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> Examples are indispensable in speculative treatises, and he who does not supply them, where they are required, raises the just suspicion that perhaps he did not understand himself. But I maintain even more, namely, that only examples from mathematics suit this purpose, since the objects of mathematics are intuitions determined in a precise fashion by concepts.
>
> (Salomon Maimon: *Kritische Untersuchungen*, Dedication to Graf Kalkreuth, GW VII, not paginated, p. VI-VII.)

1. *Introduction*

Mathematics, especially geometry, played a central role in Maimon's thought. Geometry exemplified in his eyes the best in human knowledge. His detailed discussion of various problems is of high interest in various respects. Here Maimon analyzes what "synthesis" is

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1. The work on this essay began in weekly meetings with my colleague and friend Sabetai Unguru. Besides talking about everything between Heaven and Earth, we also read texts of Maimon and discussed my interpretation of them. Sabetail also commented on my first extensive draft of this essay. Without his wide knowledge of the history of mathematics, his uncompromising acumen and his enthusiasm this essay could not have been written.

Hans Lausch (Sidney) meticulously and extensively commented on a previous draft of this essay. His suggestions were very helpful. My friend Oded Schechter read the first draft and conributed penetrating comments, Leo Corry (Tel-Aviv) and Michael Rouback (Jerusalem) also read previous drafts and offered valuable comments. I also profited from a long conversation with Daniel Warren (Berkeley), from help offered by Orna Harari and Ofra Rechter (Tel-Aviv) and from suggestions of Herbert Breger (Hannover).

My annual stays at the Max-Planck-Institute for the History of Science in Berlin are always conducive to my work. My stay there in the summer of 2005 proved especially valuable since it allowed me to read rare mathematical works of the eighteenth century. I am grateful to Jürgen Renn, the director of department 1 for the invitation and to the librarians for their great help.

Unless otherwise indicated, all translations from Maimon's works and from Hebrew are my own.

and elaborates his notion of "true synthesis." A true synthesis must produce a new object with consequences that do not follow from either of its components. Since in geometry such syntheses are due to construction and involve both the understanding and intuition, it is here that Maimon investigates the relation of understanding and intuition. Maimon argues in concreto for the impossibility of applying understanding to intuition and that, therefore, we do not have synthetic a priori knowledge in Kant's sense.[2] We have apodictic knowledge a priori of the understanding which is not synthetic, and synthetic knowledge in intuition which is not apodictic. The heterogeneity of understanding and intuition generates the "general antinomy of human thought" and finally motivates Maimon to adopt his unique "Rational Dogmatism and Empirical Skepticism."

Maimon conceives a synthetic judgment a priori as the predication of an "*idion*" in the Aristotelean tradition. Kant's key notion of "synthetic judgment a priori" did not improve on Aristotle's notion of "*idion*" and both remain obscure as long as we do not understand synthesis. In fact, Maimon's late criterion of synthetic judgments a priori is verbatim the characterization he knew of "*idion*" (*proprium* in the Latin tradition, *segula* in Hebrew), namely that it is coextensive with the "essence" of the substance but not included in its definition. For example: The definition of a human being is *animal rationale*, and *rationale* is, therefore, his constitutive property. However, every *animal rationale* and only an *animal rationale* is also an *animal riddens*. *Riddens* is hence an *idion* (*proprium, segula*) of human beings.[3] The question to be answered is *how* the *proprium* is connected to the constitutive property ("essence") of the subject. Concerning mathematics, Kant answered this question with his famous dictum

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2. Maimon developed his philosophy of mathematics in a critique of Kant. This was in line with his style of work in general: Maimon elaborated his philosophy in commentaries on other authors. On commentaries as a philosophical genre and Maimon's commentaries in particular, see my "Salomon Maimon: Commentary as a Method of Philosophizing" (Hebr.), In: Da'at 53 (2004), pp. 126-160, and my "A Philosopher between Two Cultures." In: Gideon Freudenthal (ed.): Salomon Maimon: Rational Dogmatist, Empirical Skeptic. Dordrecht (Kluwer) 2003, pp. 1-17.

   It is an irony of history that Maimon is known as a "Kantian." Maimon developed a peculiar philosophy of his won, which is opposed to Kant's in most essentials.

3. The co-extensionality of the "essence" and the "*proprium*" entails that they may exchange their places in a predication: *animal rationale* est *riddens*; *animal riddens* est *rationale*. The question, therefore, opens up how we know that "*rationale*" is the constitutive property and "*riddens*" the *proprium* rather the other way around.

that it is the construction of concepts in intuition that produces synthetic knowledge a priori:

> Philosophical knowledge is the knowledge gained by reason from concepts; mathematical knowledge is the knowledge gained by reason from the construction of concepts. To construct a concept means to exhibit a priori the intuition which corresponds to the concept. (*CpR* A 714/B742; cf. *Prolegomena* #4; AA IV, 272)

The "construction of concepts" in intuition was supposed to account for the possibility of synthetic knowledge a priori in mathematics. The definition (the concept) was supposed to imply a rule of construction by which the corresponding object is produced in intuition. Construction thus mediated between the concept of the understanding and intuition.[4] Note that concept means here "definition," not "empirical concept" or "picture." We may recognize by sight the "roundness" of a plate, say on the basis of its similarity to other pictures. But Kant says that "the empirical concept of a plate is homogeneous with the pure geometrical concept of a circle. The roundness which is thought in the latter can be intuited in the former." (*CpR* A137/B176) However the geometrical propositions establishing the properties of the circle are not "read off" the plate but, on the contrary, apply to the plate only if its roundness conforms to the geometrical definition of the circle. If this is not so and we merely recognize the roundness in intuition, then roundness is not "thought" at all, but is merely a perception or a "picture" and we may not apply our geometrical knowledge to this object. How, then, do we construct a concept in intuition?

Since Euclid's geometry constructs all its objects from a straight line and a circle, which are accepted as primitives, Kant's thesis on constructing concepts in intuition can be tested here: Do we construct also these primitive concepts in intuition and, if so, does construction

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4. Kant also uses the term "scheme" for this mediating function: "This representation of a universal procedure of imagination in providing an image for a concept, I entitle the schema of this concept.

Indeed it is schemata, not images of objects, which underlie our pure sensible concepts. No image could ever be adequate to the concept of a triangle in general. It would never attain that universality of the concept which renders it valid of all triangles, whether right-angled, obtuse-angled, or acute- angled; it would always be limited to a part only of this sphere. The schema of the triangle can exist nowhere but in thought. It is a rule of synthesis of the imagination, in respect to pure figures in space." *CpR* A140-141/B179-180.

See the interpretations of Friedmann 1999, 124-125 and Koriako 1999, # 18, pp. 222-237.

explain how synthetic knowledge a priori is possible? The latter question has been widely debated in recent decades but will not be discussed here. The problems Maimon addresses precede all "enunciations" of geometrical propositions and all "auxiliary constructions" introduced to prove them.[5] Maimon's objections refer in the first place to the possibility of constructing the two basic elements of all further constructions - the straight line and the circle - and to two examples of synthetic a priori judgments, which are not proven in Euclid's Elements but referred to by Kant: The straight line is the shortest between two points;[6] A tri-

5. Jaako Hintikka argued in a series of papers that in Euclid some geometrical arguments cannot be carried out without auxiliary constructions and that this is the basis of their synthetic nature according to Kant.

In his "Kant on the mathematical method", The Monist, 5I, (1967), 352-375, reprinted in Kant's Philosophy of Mathematics. Modern Essays. Edited by Carl J. Posy, Dordrecht etc. (Kluwer) 1992, pp. 21-42, and in later papers, Hintikka focues on Kant's concept of construction. Hintikka suggested that construction "is tantamount to the transition from a general concept to an intuition which represents the concept" (21) and that "Intuitivity means simply individuality".(23) Intuitions, says Hintikka, are "particular representatives of general concepts" (33) or, in contemporary logical parlance, "existential instantiations". (35). However, "...within geometry Kant's notion of construction coincides with the ordinary usage of the term 'construction'" (30), more precisely with Euclidean 'Echthesis' (29-30). (See also See Jaako Hintikka, "Kant's theory of Matheamtics Rivisited", in: Philosophical Topics. Volume 12, Number 2: Essays On Kant's *Critique of Pure Reason*, 201- 215). And yet, neither Hintikka nor later participants in the ensuing debates discusses this very "transition from a general concept to an intuition which represent the concept" and how we ascertain that the intuition indeed corresponds to the concept. An exception is Judson Webb's "Immanuel Kant and the Greater Glory of Geometry", in: Naturalistic Epistemology, ed. by Abner Shimony and Debra Nails, Dordrecht etc. (Reidel) 1987, pp. 17-69, esp. 22-27. Webb notes that we have no rule of construction for the straight line (20) and the different situation in the case of the circle, where a rule (involving motion) was available. Webb seems to accept Kant's position that this rule is a corollary of Euclid's definition of the circle. This assumption was contested by Maimon and will be discussed below. However, these Euclidean postulates and Kant's claims concerning them are not further discussed by Webb.

Be it as it may, the proposition "The straight line is the shortest between two points" which served Kant as an example for synthetic a priori judgments in geometry cannot be interpreted according to Hintikka's suggestion (see below # 2). The same holds for another example of Kant, namely "figura trilateris est triangularis" (see below # 4.2). Both propositions are neither axioms or postulates nor are they proven by auxiliary constructions.

6. Michael Friedmann argues that iterative constructive processes and motion served Kant to ensure the density and continuity of geometrical objects which monadic logic cannot provide. (Friedmann 1992, chapter one, 55-95). Kant "appears to be echoing" Newton's "fluxions", i.e. the use of motion to create continuous curves. (72-75) Concerning the proposition that the straight line is also the shortest between two points, Friedmann suggests that Kant may have thought of Euler's variational methods for proving geodesicity and adds the "speculation" that "synthetic" here refers to the integration involved in these methods. (Friedmann, 87, note 54). However, Kant never mentions a proof for this synthetic proposition, nor is there anything in what we know about

lateral figure has three angles. These pivotal examples are not discussed by contemporary interpretations of Kant's philosophy of mathematics.

Maimon showed that for these most important instances, the straight line and the circle, the rule of construction is not implied by the concept of the object. In the case of the straight line, we simply have no rule for its construction; in the case of the circle we have a rule which is neither identical with the definition nor implied by it. Thus we construct a "circle" according to a rule, but we do not yet know that the constructed object corresponds to the definition of the circle. This has to be subsequently proven. This duality of definition and construction is one form of the unbridgeable gulf between understanding and intuition. Kant's thesis that geometry is synthetic a priori due to the construction of concepts in intuition thus lost its fundament. Much of Kant's philosophy is built on this fundament.

The thesis that there are synthetic judgments a priori in mathematics can apply to its foundations - axioms and postulates - or to propositions and their proofs, or to both. I will argue below that Maimon first attempted to show that Kant's paradigmatic example of synthetic judgments a priori can be reduced to analytic judgments and, failing to do so, he later argued that this example and other (but not all) propositions of geometry are dependent on intuition, and therefore synthetic a priori, but not pure nor necessary, but "imposed" on our intuition. Maimon thus consistently rejected Kant's explanation of the possibility of truly necessary synthetic judgments a priori, namely the construction of concepts in intuition, and he was not willing to accept intuition (a priori or a posteriori) as a legitimate source of genuine knowledge. Necessary judgments are to him explications of concepts or of their defnitions (Tr 380-386).

It may hence turn out that the *proprium* is implied by the definition of the subject term and the alleged synthetic judgment a priori is in fact analytic; it may turn out to be synthetic a posteriori - and its alleged necessity nothing but psychological compulsion, or, finally, it may be genuinely synthetic a priori, i.e. synthetic but involving only conceptual knowledge without recourse to intuition.

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his knowledge of mathematics that suggests that he was aware of Euler's variational methods or capable of understanding them.

Maimon draws from this experience an important philosophical consequence concerning the notion of a mathematical or philosophical "system." Whereas Kant attempted to construct *ab ovo* also the primitive objects which Euclid introduced by postulates (the straight line and the circle), Maimon rather maintained that in all knowledge we begin and end "in the middle" (Tr, 350). We can reach neither rock bottom nor the peak of our conceptual systems.

In geometry, Maimon therefore suggested either to accept, as Euclid did, the straight line and the circle as primitive objects introduced by postulates, or to construct these primitive objects by means of conic sections. Needless to say that the construction of a cone presupposes the existence of a circle and a straight line. I therefore maintain that Maimon's entire philosophic project is fundamentally opposed to Kant and even more so to Post-Kantian philosophy. The latter wished to construct the entire fabric of knowledge from a self-evident first principle. This Post-Kantian program can be interpreted as a radicalization of Kant's idea of construction which, in its turn, radicalized Euclid's idea. Whereas Euclid was content to construct all geometrical objects from the straight line and the circle which were accepted as given by postulates, Kant wished to construct even these primitive objects with nothing but a moving point. Maimon moves from Kant in the direction opposed to that taken by German Idealism. He is willing to begin in the middle, with complex objects - here: the cone- and further determine this general concept by *differentiae specificae* and thus generate individual objects: circle, ellipse, hyperbola, and parabola, indeed even the straight line and the point.

This is also the model he follows in philosophy. We begin in the "middle" and work our way up to the most general concepts as well as down to the most particular. But as finite intellects, we cannot reach the extremes. Maimon reached these insights concerning geometry and philosophy after a zigzag course in which he radically changed his views more than once. I will now review in brief the most important stations of this path and the reasons for Maimon's abrupt changes.

1.1. *A Failed Proof and a Philosophical Conversion*

In 1792 Salomon Maimon confessed in a casual footnote that in his *Versuch über die Transcendentalphilosophie* (1790) (Hereafter: Tr) he attempted a "salto mortale". This was the unification of Kantian philosophy with Spinozism.[7] Now he was "perfectly convinced" that this project could not be realized and rather wished to unite Kantian philosophy with Humean Skepticism. (GW III, 455)

This short remark is amazing for a number of reasons. First, the liaisons mentioned seem quite awkward in themselves. What would count as a successful synthesis of Spinoza and Kant or Hume and Kant? Would not such a combination of Kantianism and either rationalism or skepticism be inconsistent? And was it not Kant's claim that his critical philosophy opened a new venue and superseded the alternative between "rational dogmatism" and "empirical skepticism"? Finally, more or less at the same time, Maimon characterized his philosophy as "Rational Dogmatism and Empirical Skepticism" (Tr 436; GW I, 558), hence rather as a combination of Spinoza (or Leibniz) with Hume than with Kant. What was hence Kant's role in this newly adopted philosophical position? And what was the reason behind such a radical change from Kant-Spinoza to Kant-Hume? Moreover, Maimon's remark is also astounding because we do not know of any such radical change in his thought after the publication of his Transcendentalphilosophie. The skeptical positions associated with the name "Hume" are presented right there, in the *Transcendentalphilosophie* itself, and Hume is also explicitly and repeatedly named in this book and even in Maimon's letter to Kant which accompanied the manuscript of *Transcendentalphilosophie* sent to him.[8] Furthermore, in the years following the publication this work, Maimon continued to express the same "Spinozistic" views to be found in the book and overtly sided with Spinoza against his critics. In short: There is no trace of a radical change in Maimon's views concerning either Hume or Spinoza

7. This is one of the footnotes Maimon added as the editor of Magazin für Erfahrungsseelenkunde to the paper "Obereits Wideruf für Kant" published in this journal (Vol. IX, No. 3 (1792), pp. 106-143; GW III, 418-455). Jacob Hermann Obereit (1725-1798) himself attempted here a combination of Kantianism (in the interpretation of Reinhold) and Spinozism (in a highly confused enthusiastic prose and indebted to Mysticism which he also endorsed in his other writings). The expression "salto mortale" has at the time "Spinozistic" connotations. Jacobi introduced it to refer to his leap from Spinozism to faith, or rather from any rational metaphysics in general to faith (Scholz, 81, 91). The expression was taken up by Mendelssohn in his polemics against Jacobi (Scholz, 114), and it was used by Obereit in the text discussed here (GW III, 454).

8. See Tr, 70-74. Note that this discussion is conducted on the four last pages of this chapter. I argue below that they were added later. See Maimon's letter to Kant, April 7, 1789, AA XI, 15-17.

after the publication of the *Transcendentalphilosophie*.

I offer one and the same explanation for these observations: In the course of editing the *Transcendentalphilosophie* Maimon discovered that he had failed in his attempt to refute Kant and establish that geometry is analytic. The issue was specifically the proposition that the straight line is also the shortest between two points. This was the example from geometry with which Kant introduced the claim that geometry was synthetic a priori. Maimon attempted to prove this proposition and hence show that it is analytic. If successful, Maimon's proof would have undermined Kant's claim that there are synthetic judgments a priori in geometry and demonstrated that intuition was not essential to the enlargement of our knowledge. It would have refuted an argument for Kant's claim that understanding and intuition are independent from one another, reduced in this case intuition to the understanding and supported the call to reinstall Leibnizian philosophy (also called Spinozism by Maimon).[9] Indeed, the failure of Maimon's proof was unambiguous and justified the expression that he became "perfectly convinced" that his philosophical program failed.

I will later discuss the proof in extenso. Here it suffices to say what the discovery was that undermined the proof. It was that a curved and a straight line are conceptually disjunct although a curve may be infinitely approximated by a broken straight line. Now this discovery also showed (pace Kant) that in the case of the circle - one of the two primitive objects in Euclid - the definition does not imply a rule of construction. When used as a rule of construction, the definition of a circle produces a polygon and not a circle. Although the circle can be approximated by the polygon, they remain conceptually disjunct because a circle is a curved line whereas the polygon is a broken straight line.

The crucial importance of this issue for Maimon is not surprising, for he conceived mathematics as the (only) exemplification of human knowledge strictu sensu. However, whereas Kant referred to the "fact of science," of mathematics and pure physics, to ground his thesis that there are synthetic judgments a priori (*CpR* B 19-21 and B 128; *Prolegomena* #4; AA IV, 275), Maimon doubted this very fact, initially because he believed that all true

9. Maimon refers to the paramount importance such a proof a few years later: "The dependency of sensibility on the understanding ist not proven in the Wolfian-Leibnizian philosophy but it is merely assumed that sensibility only represents in an obscure way what the understanding must think of the things in themselves in an explict way" (V, xviii)

judgments can be reduced to analytic judgments, later because he believed that they might not at all depend on the understanding but on intuition alone (and, therefore, not be a priori in Maimon's sense), perhaps even empirical. From here two possibilities opened up: to attempt to prove the necessity of truths of mathematics (thus working towards "Dogmatic Rationalism"), or to accept that they are merely empirically (or subjectively) true ("Empirical Skepticism"). Since a choice between the two alternatives could not be justified, Maimon simultaneously maintained the possibility of both. This is Maimon's unique combination of Dogmatic Rationalism and Empirical Skepticism. An intermediate position is also elaborated: it is Maimon's Law of Determinability which can detect categorical mistakes in predication or show which predications are well-formed. It can thus serve as a criterion of "possibility," not of truth.[10]

This change of mind from Rationalism to Maimon's mature position occurred when Maimon edited the manuscript of his *Versuch ueber die Transcendentalphilosophie* in the Autumn of 1789. Maimon's book is famous for its lack of order and he himself admitted that he had not succeeded in making it cohere. Since he did not delete (all?) passages expressing his former view, the book contains irreconcilably contradictory views on the possibility of dispensing with intuition. Thus Maimon's point of departure is that the proposition "The straight line is the shortest between two points" is analytic and a priori, whereas later he maintains the extreme opposite, namely that the proposition is synthetic a posteriori and based on experience . On yet another occasion he maintains that the proposition is indeed a priori and synthetic, but that the reasons for its truth are obscure and that it is imposed on our intuition, not accepted by our understanding.

We can even reconstruct the most important circumstances of this rewriting and determine that Maimon made his discovery just before the publication of his Transcendentalphilosophie. More important than the exact date, is the fact that many passages of *Transcendentalphilosophie* show that Maimon edited the text in response to Kant's critique of the manuscript. If one considers that he edited his manuscript in the few months between the receipt of this letter and going to print and while his thought was undergoing a radical change,

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10. For the claim that Mathematics, especially geometry, is a model which accords with the Law of Determinability see GW I, 125.

this would explain why he did not succeed in revising the book from the point of view of his new position.

I therefore submit that when Maimon said that in the *Transcendentalphilosophie* he attempted to join Kantianism and Spinozism he meant the original manuscript (which was sent to Kant), and when he said that he later changed his mind, he meant the second phase, when he revised and rewrote the book immediately before its publication at the end of 1789. This reconstruction is strongly supported by the fact that there is indubitable textual evidence independent of my interpretation that Maimon replied to Kant's critique in the printed version of the book.[11] Note finally, that interpreting the expression "*Versuch über die Transcendentalphilosophie*" as referring exclusively to the original body of the manuscript and not also to the notes (and the synopsis) is supported by Maimon's usage in the *Transcendentalphilosophie* itself! In his endnotes to Tr, Maimon refers to an argument developed in the main body of the text with the words "I have remarked already elsewhere" and the reference given in the footnote is: "Versuch über die Transcendentalphilosophie" - as if the endnotes were not part of the same book! (See Tr, GW II, 400. The original pagination is wrong here.)

The thesis that the discovery of his failure to render mathematics analytic was the reason for revising his initial philosophical program is further strongly supported by Maimon's paper "Answer to the previous letter" published only a few months after the *Transcendentalphilosophie*. Asked by the editor of the "Bernlinisches Journal für Aufklärung", Andreas Riem, to succinctly explain what the gist of his book was, Maimon illustrated its various arguments with the same mathematical example that was involved in his discovery that the attempted proof failed.[12] In the light of the evidence, I believe that both his "conversion" and the fact that no such conversion occurred after the publication of Transcendentalphilosophie can be explained. Much more important is however the insight this episode provides into Maimon's philosophy of mathematics and into his resulting unique philosophy of "dogmatic

11. It can be shown that Maimon made his discovery after receiving Kant's letter and before the publication of his *Transcendentalphilosophie*, hence some time between July 1789 and the printing of the book in December 1789. See the appendix to this essay.

12. Antwort des Hrn. Maimon auf voriges Schreiben. In: Bernlinisches Journal für Aufklärung. 1790, Bd. IX/1, 52-80. Valerio Verra identified the addressee as Andreas Riem, the editor of the journal. (See GW VII, 722)

rationalism and empirical skepticism".

1.2. *The Value of Mathematics*

Kant introduced his claim that there are synthetic a priori judgments with reference to two realms where such judgments were allegedly a fact, such that only its possibility needed to be explained. Maimon's doubted this fact in both these realms: in mathematics, especially geometry, and in mathematical physics. However, there is more at stake for Maimon than the successful grounding of mathematics. In mathematics we not only think about existing objects but construct the objects by means of thought. In this we resemble God in whom thought and creation are one.

> All concepts of mathematics are thought and simultaneously presented by us as real objects by means of a priori construction. In this we are similar to God. (GW IV, 42).

Matheamtics and other "speculative sciences" (i.e. philosophy) testify to the "divine spark" present in human thought and to the possibility of attaining perfection.[13] Mathematics

13. "Was soll man nun von Weltleuten, ja sogar von Gelehrten denken, welche die spekulativen Wissenschaften bloß darum verachten, weil sie keinen unmittelbaren Nutzen im gemeinen Leben haben? Was würde ein Neuton (sic!), ein Leibnitz dazu sagen, wenn sie hören sollten, daß man ihre herrliche Erfindung (Differentialrechnung) nicht als einen Funken der Gottheit, als einen Adelsbrief, wodurch die hohe Abstammung des menschlichen Geistes von den reinen Intelligenzen bewiesen wird, sondern bloß des Nutzens wegen schätzen will, daß man dadurch (in der Artillerie) berechnen kann, wie man die größte mögliche Anzahl Menschen in der kürzesten Zeit tödten kann? Wer kann die Ausübung der Seelenkräfte an sich, sollte sie auch keinen andern Nutzen haben, unnütz nennen? und wer kann die mit dieser Ausübung verknüpfte Glückseligkeit wegraisonniren? Gewiß nur der, der sie nie genossen hat. ... Ich habe ausgeschweift, aber es war ein Wort zu seiner Zeit. (*Versuch einer neuen Logik*, 1794, GW V, 266f) In his obituary on Maimon, Lazarus Bendavid tells the following anedcote on what happened when Maimon's was finally admitted to Berlin in 1780: "Nach Tische nahm ich ihn mit auf mein Zimmer; und da er mir sagte, daß die Absicht seiner Reise nach Berlin bloß wäre, Wissenschaften zu treiben, zeigte ich ihm einige mathematische Bücher, aus denen er mich bat ihm einige Sätze vorzulesen. Ich thats; aber nie war ich so erschüttert als damals, da ich Thränen aus seinen Augen fließen sah, und ihn laut schluchzen hörte, O, mein Sohn, sagte er mir weinend, wie glücklich bist du, so jung die Werkzeuge zur Vervollkommnung deiner Seele zu haben und gebrauchen zu können. Herr der ganzen Welt! Ist Erlangung der Vollkommenheit Bestimmung des Menschen; so verzeihe mir die schwere Sünde, wenn ich frage, warum mir Armen bis jetzt die Mittel benommen waren, meiner Bestimmung treu zu leben. - Er bedeckte sich das Gesicht mit beiden Händen und weinte bitterlich."(Lazarus Bendavid, Über Salomon Maimon, in: National-Zeitschrift für Wissenschaft, Kunst und Gewerbe in den Preußischen Staaten, Bd. 1 (1801), S. 88-104, here: 93)

stands in Maimon's philosophy for the possibility of basing human knowledge on pure understanding, of dispensing with sensuality and imagination which are dependent on corporeality and elevating Man to the company of the angels and God rather than associating him with the beasts. If, on the ohter hand, human cognition is but sensual, then it is not specifically different from the "knowledge" of brute animals, nor is Man an "*animal rationale*": " ... so that a man hath no preeminence above a beast ..." (Ecclesiastes 3, 19)

Rationalism therefore had great ethical value for Maimon. Pure thought (as it is exemplified e.g. in algebra) manifests in Maimon's eyes Man's essence as an *animal rationale* and testfies to Man's divine origin. Maimon's mature philosophical position - the ever-present alternative between his own versions of Rationalism and Skepticism - is also the alternative between Man's kinship with the divine or with the beasts.[14] There is no doubt where Maimon's sympathies lie. It speaks for his intellectual integrity that he did not deceive himself into believing that he could substantiate Rationalism (or Empiricism) in a satisfactory way and that he suspended judgment in this vital question.

Indeed, we see that at least since Maimon's arrival in Berlin in 1780 onwards, Mathematics played an imporant role in his philosophical thought, but not less so in his engagement in the Jewish Enlightenment and in his personal life. Maimon considered mathematics a major vehicle to human perfection and was moved to tears when he first experienced in Berlin the free access to textbooks of mathematics.[15] He recounts with pride of his own achievments in mathematics when he attended as an adult a secondary school for two years.[16]

These views and experience are the background of Maimon's discussion with Mendelossohn and others over the ends and means of Jewish Enlightenment. While some representatives of the Enlightenment believed that the rational critique of the religious views of the

14. When Maimon Introduced the idea that the human intellect is, although limited, the same in kind as the Divine "infinite intellect," he added that this Idea was sublime (erhaben) (Tr, 65). I know of no other occasion on which Maimon uses this word.

15. Lazarus Bendavid: Ueber Salomon Maimon. In: National-Zeitschrift für Wissenschaft, Kunst und Gewerbe in den preußischen Staaten: nebst einem Korrespondenz-Blatte 1, 1801, 88-104, here: 93-94.

16. GW I, 533-534. Maimon's self-praise seems at least partially confirmed by the remark "optimus Judaeorum" in the entry "Salomon Maimon" in the register of the "Christianaeum."

Jews was the task to be pursued, Maimon believed that the diffusion of mathematical and scientific knowledge was more important, and he indeed wrote Hebrew textbooks in mathematics and physics. However, in our following remarks Maimon's philosophy of mathematics proper is at stake, although we should bear in mind that its significance and importance for Maimon depended also on its role in his philosophy in general and in his social and individual life.[17]

2. *The Straight Line*

2.1. *Synthetic Judgments a priori Kantian and Aristotelean Style*

"The general Problem (Aufgabe) of Pure Reason" says Kant under this heading in the second edition of the *Critique of Pure Reason* is to answer the question: "How are synthetic judgments a priori possible?" (*CpR*, B 19)

That such synthetic judgments a priori are real is proved by the fact (Factum) that "pure mathematics and general natural science" exist (*CpR*, B 128).

Thus the answer to the question how such judgments are possible also answers the following questions (*CpR*, B 20):

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17. The central role of mathematics in Maimon's life and thought, especially in his philosophy, is no secret. And yet, there is as yet no satisfactory study of his views of mathematics. The few treatments extant do not elaborate Maimon's discussions of particular problems, and thus, I believe, miss Maimon's essential tenets. See Shmuel Hugo Bergman, The Philosophy of Solomon Maimon. Translated from the Hebrew by Noah J. Jacobs, Magnes Press: Jerusalem, 1967, chapter 7. David R. Lachterman, "Mathematical Construction, Symbolic Cognition and the Infinite Intellect: Reflections on Maimon and Maimonides", Journal of the History of Philosophy, 30, 4, (1992) pp. 491-522. This is an important, but obviously not completed study. Meir Buzaglo, Salomon Maimon: Monism, skepticism, and Mathematics. Pittsburgh, Pa.: University of Pittsburgh Press, 2002. This is a collection of rather loosely connected reflections on various philosophical problems of mathematics apropos of Maimon. The most detailed and recent study is: Christian Kauferstein, Transzendentalphilosophie der Mathematik. Versuch einer systematischen Rekonstruktion der Leitlinien einer Philosophie der Mathematik in Kants 'Kritik der reinen Vernunft' und in Maimons *Versuch über die Transcendentalphilosophie*' Philosophische Dissertation, Giessen 2004. Kauferstein "willingly" hazards the consequences of neglecting "examples" in order to enable a "general consideration" of Kant's and Maimon's philosophy of mathematics. (p.11, 30-32) I argue in this essay that Maimon philosophy of geometry (at least) evolves out of his study of the two "examples": The straight line and the circle. Very insightful is a short chapter in Richard Kroner, Von Kant bis Hegel, 2 vols., (1921,1924), Tübingen (J.C.B. Mohr) I, 344-353

> "How is pure mathematics possible?
>
> How is pure natural science possible?"

And Kant continues:

> "...since they are actually given, it can appropriately be asked how they are possible; for that they must be possible is proved through their actuality (Wirklichkeit)". (*CpR*, B 20-21).

The claim that there is a synthetic a priori part of physics (physica pura) was later (and in my view: successfully) criticized by Maimon.[18] Here we are rather concerned with his criticism of the thesis that mathematics is synthetic a priori.

Kant brings two examples to substantiate this de facto claim, one for arithmetic, the other for geometry. The example from arithmetic is: 7+5=12. The paradigmatic example from geometry is the proposition that the straight line between two points is also the shortest between them.

> That the straight line between two points is the shortest is a synthetic proposition, for my concept of the straight contains nothing of quantity, but only a quality. The concept of the shortest is therefore entirely additional to it, and cannot be extracted out of the concept of the straight line by any analysis. Help must here be gotten from intuition, by means of which alone the synthesis is possible." (*CpR* B 16)[19]

It is clear that if the claims that there are synthetic judgments a priori is refuted, then their possibility need not concern us.[20] Since his claim that there are synthetic a priori judgments

18. See my "Maimon's Subversion of Kant's *Critique of Pure Reason*: There are no Synthetic a Priori Judgments in Physics."

19. "Daß die gerade Linie zwischen zwei Punkten die kürzeste sei, ist ein synthetischer Satz. Denn mein Begriff vom Geraden enthält nichts von Größe, nur eine Qualität. Der Begriff des Kürzesten kommt also gänzlich hinzu und kann durch keine Zergliederung aus dem Begriffe der geraden Linie gezogen werden. Anschauung muß also hier zu Hülfe genommen werden, vermittelst deren allein die Synthesis möglich ist."(KrV B 16)

20. Concerning the dependence of Kant's Critique on the quid facti, see also Maimon, IV, 210-211, 225-226, 229

was substantiated with examples, their refutation was all that was needed to ward off Kant's major claims. It is now clear why the success or failure to prove that the straight line is also the shortest between two points is crucial to Kant's philosophy and to its Leibnizian alternative: If from the definition of a straight line and the preceding geometrical definitions, axioms and postulates it can be inferred that it is also the shortest between two points, then the proposition is rendered analytic and Kant's claim to the contrary fails with all its consequences. If the proof fails, then Kant's claim gains high plausibility although it is not proven. In the latter case, Kant's question "How are synthetic judgments a priori possible" is at least meaningful and cannot be ignored. This context explains why Maimon's attempt to prove that the straight line is shortest between two points is crucial to a well-founded judgment on Kant's claim to have revolutionized philosophy.[21] However, this context should not blind us to the fact that the issues to which Kant refers were well known at least since Antiquity. I will show this in brief for both the geometrical and the general philosophical problem involved. Geometry first.

Diogenes is reported to have mocked geometers for their attempt to prove the proposition that the straight line is the shortest between two points. He said that even a donkey knows that the straight way to the fodder is also the shortest and hence chooses it[22]. Indeed,

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21. Buzaglo bases his main theses ("Maimon's Ladder", pp. 49-76) on Maimon's claim that he can reduce the "straight line between two points" to the "shortest" line between the same points. However he discusses neither Maimon's proof nor its failure. Moreover, Buzaglo's interpretation of "Maimonic Reduction" depends mainly on a discontinued quotation. He translates: "If an [sic! instead of "my") intuitional mode were eliminated, there would not be any intuition nor any objects of thought which are determined in and of themselves". (Tr 206; Buzaglo 61). From here he concludes in Maimon's name that without intuition there would be no objects of thought. However, the continuation of the very same sentence reads: "But since my faculty of thought could still endure, it could still generate objects of thought out of itself (ideas which become objects determined by thought), because I maintain that the connection of thought with the faculty of intuition in general, and not merely with a specific one, is merely contingent..." Buzaglo's thesis that even the objects of mathematics and pure concepts are dependent on intuition (Buzaglo 61-62) is the very opposite of what Maimon explicitly says here (and elsewhere).

22. Eduard Zeller, *Die Philosophie der Griechen in ihrer geschichtlichen Entwicklung*, 4. Aufl. Bd. II,1: 289, n.2, quoting Simplicius, *De Coelo* 33b, Schol. in Aristotelem 476b 35. See also Heath I, 278. See Mendelssohn's allusion to this tradition: “Der Geometer entsieht sich nicht, nach der Strenge zu beweisen, daß die gerae Linie der kürzeste Weg zwischen zwey Punkten sei; ob ihm gleich der Cyniker mit Recht vorhält, daß dieses auch dem Hunde bekannt seyn müsse, der seinen Raub in gerader Linie zu ereilen sucht.” Moses Mendelssohn, *Morgenstunden*, sechste Vorlesung, (JubA III.2: 50)

while some mathematicians attempted to prove the proposition, Archimedes introduced it as a postulate:

> "Of all lines which have the same extremities the straight line is the least."[23]

If the property "shortest" is not proven (and thus reduced to "straight"), the philosophical problem posed by this geometrical example is this: How are we to understand that the judgment "The straight line is the shortest between two points" is necessarily true although it is synthetic? This is the Kantian garb of the problem. The same problem appears in Aristotelian vestments in the following manner: How are we to understand that whenever the constitutive property ("straight") applies to a line also another property ("shortest") applies?

In general terms, the problem is that in addition to the essential property defining a substance, there may be another or some other properties that are co-extensive with the essential property but which are not part of the definition of the substance. This co-extensionality is, therefore, not understood. Aristotle named such a property "idīon." Thus, whenever and only when the essential property (the *differentia specifica*) - and therefore the substance - is present, this additional property is also present. For example, the human being is defined as *animal rationale*. "*Animal*" is the genus and "*rationale*" the defining essential property of humans. However, whenever "rational" is present, the ability to "laugh" is also present. We do not define humans as "*animal riddens*", although "*riddens*" and "*rationale*" are co-extensive. The same observation applies to "*sociale*" as well. Moreover, we could even refer to bodily

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Kant's opponent, Johann August Eberhard, showed little understanding for the problem involved:

"Man sagt mit Recht: der gesunde Verstand lehrt, die kuerzeste Linie zwischen zwei Punkten ist die gerade. Dieses einzusehen, dazu ist keine Demonstration durch viele Schluesse noethig." See Johann August Eberhard's Synonymisches Handwoerterbuch der detuschen Sprache, (1802), 5. Aufl. (1821) # 1204: Vernunft. Verstand, Urtheilskraft. (p. 654)

23. Archimedes, *On the Sphere and the Cylinder*, Postulate 1. *The Works of Archimedes*, translated and edited by Thomas L. Heath (1897), New York (Dover) 1953, p. 3. Reviel Netz translates: "That among lines which have the same limits, the straight [line] is the smallest." *The Works of Archimedes*, Cambridge (Cambridge University Press) 2004, p. 36. For Eutocius' proof of the proposition, see Netz, 245-246.

See also Louis Couturat, "Kants Philosophie der Mathematik" (originally published in Revue de Métaphysique et de Moral, May 1904), in: *Die philosophischen Prinzipien der Mathematik*, Leipzig (Klinkhardt) 1908, 247-326, esp. 292-296.

properties which singularly characterize humans and certainly do not define their essence nor seem connected to this essence: having broad fingernails or a wide chest, being bipedal. Such properties are located as it were between the essential *differentia specifica* and the accidents. They are co-extensive with the *differentia* and like the accidents they are not part of the definition of the essence. In the Latin tradition such a property was called "*proprium*", and in the Hebrew tradition "*segula*".[24] Of special interest is that Aristotle names as an example for an "idīon" the property that the sum of the interior angels of a triangle equals two right angles. This property is proven in a geometrical proposition in Euclid's *Elements* and will be discussed later[25] The example is of great interest because it suggests that there is an intrinsic connection between the essential properties of the object and the *proprium*. There can be no doubt that whether part of the definition or not, the sum of the angles in the triangle is closely connected to its "essence" and is not merely an accidental property of all triangles. But does the definition of the triangle imply that the sum of its internal angles is equal to two right angles? If it does, the proposition is analytic, if not: synthetic. But if it is synthetic, what is the nature of the connection between the essence and the *proprium*? And how do we know that "straight" is the constitutive property and "shortest" the *proprium* and not the other way around?[26]

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24. See Maimonides *Millot ha-Higayyon*, chapter 10. Shmuel Ibn Tibbon: *Bi'ur Hamilim Hazarot* under "Ichut". See also Klatzkin, *Ozar Hamnuachim Haphilosophiyim*, vol. IV, pp. 60-62; see also the discussion of *segula* in: Dov Schwartz, *Amulets, Properties [segulot] and Rationalism in Medieval Jewish Thought* (Hebr.), Bar-Ilan University Press, Ramat Gan 2004, pp.107-113 129-135. In the Middle Ages *segula* also acquired the meaning of a "*qualitas occulta*". See below on Mendelssohn's view of the *Segula*.

25. *Topics*, I,4, 101b17-23; I, 5, 101b37-102a2, I, 5, 102a18-22. In the Metaphysics Aristotle once names this property "accident": "'Accident' has also (2) another meaning, i.e. all that attaches to each thing in virtue of itself but is not in its essence, as having its angles equal to two right angles attaches to a triangle." (Topics V,29; 1025a 30-35). A somehow more elaborated discussion of these properties is given in De anima, but the same example is repeated there: "It seems not only useful for the discovery of the causes of the derived properties of substances to be acquainted with the essential nature of those substances (as in mathematics it is useful for the understanding of the property of the equality of the interior angles of a triangle to two right angles to know the essential nature of the straight and the curved or of the line and the plane) but also conversely, for the knowledge of the essential nature of a substance is largely promoted by an acquaintance with its properties" (*De anima* I,1; 402b 17- 403a 3).

26. In the very influential edition of Euclid, in which two translations, one from the Greek, the other from Arabic, were printed together, the translation from the Greek defines the line as the shortest: "Linea recta, est ab uno puncto ad alium breuissima extensio, in extremitates suas eos recipiens"

The concept of these properties was also discussed by Maimonides in his logical treatise *Millot ha-Higayyon*, a copy of which - with Mendelssohn's commentary - was in Maimon's possession (GW I, 457). Maimon himself discusses the notion of *Segula* in his commentary on Maimonides' *Guide of the Perplexed*.[27] In the same commentary he also refers to the proposition that the straight line is the shortest between two points as *segula*. He even may have been inspired at this point by Maimonides himself.[28]

In the tenth chapter (§ 6) of his *Millot ha-Higayyon* Maimonides defines the *Segula* in contradistinction to the *differentia* and to the accidens:

> "A *differentia* is that by which one genus is recognized and distinguished from the other and is what constitutes the thing. And what is always found in all individuals of the genus, and does not constitute that genus is called *segula*, and that which is found in many or few of [the individuals of] the genus and does not constitute it, is called accident.[29]

In his commentary, Mendelssohn points out that the coextensionality of essence and *proprium* does not permit one to determine with certainty whether a property belongs to the essence of the substance or is merely a *proprium*:

(p.1)), whereas the translation from the Arabic has a variant of the traditional definition ("Recta linea, est quae ex aequali, sua interiacet signa" (p. 4). See *Euclidis megarensis mathematici clarissimi Elementorum geometricorum*, lib. XV: cum expositione Theonis in priores XIII Bartholomaeo Veneto latinitate donata, Campani in omnes, & Hypsiclis Alexandrini in duos postremos ; his adiecta sunt Phȋnomena, Catoptrica & Optica, deinde Protheoria Marini & Data, postremm uero opusculum de leui & ponderoso, hactenus uisum, eiusdem autoris (1537; and many more editions). See on this edition: Heath, I, 98.

27. See *Givat Hammore*, p. 21-22. See Gideon Freudenthal and Sara Klein-Braslavy: "Salomon Maimon reads Moses Ben-Maimon: On Ambiguous Names", Tarbitz, Vol. 72, No. 4 (2005), pp. 1-33.

28. Maimonides discusses in *Guide* I, 52 different kinds of properties, one kind of which are those of quantity. The example he gives is astonishing: "long, short, crooked, and straight and other similar things." Pines, 116. Maimonides seems to suggest that "crooked" and "straight" are (essentially?) quantities like "long" and "short", not qualities. I do not know whether this was an oppinion held by Arab mathematicians.

29. In chapter VIII of his commentary to Tract Abbot, Maimonides names free will as a *Segula* of Man.

Segula is specific to the individuals of one genus and is not found in [those of] another, nor is there an individual of that genus which lacks that *Segula*, but it does not constitute this entity (davar) and its truth, and we do not know whether in its absence that genus is also absent, and therefore we give it a specific name and call it *Segula*. And such is the laughter in man as mentioned by the Rabbi [Maimonides], and that the spider weaves its web and the bee produces honey and hexagonal chambers, and the magnetic stone attracts iron and many more of this kind, since we see that all individuals of that genus have this *Segula*, but we do not know whether it belongs to the essence of this entity or whether e.g. the bee would not any more be what it is if it were deprived of producing honey."[30]

Another novelty comes in the Mendelssohn's commentary on § 10. Maimonides specified there that each genus necessarily has one or several *segulot*, and Mendelssohn comments:

"The reason for this is that even though due to the shortage of our understanding we do not know what the connection is between the *segula* and the genus and why it is found in all individuals of one genus and not in those of another, there is nevertheless no doubt that it is to some end that the Creator, may He be blessed, attributed this *segula* exclu-

30. This and the following quotation: Moses Mendelssohn, *Bi'ur Millot ha-Higayyon le-Hanesher ha-Gadol Rabbeinu Moshe Bar Maimon, zts"l*, 1765, commentaries on # 6 and 10, JubA XIV (1938), S. 88-90

The word "*segula*" is used in the Pentateuch to designate the property of the people of Israel, singled out by God's choice (e.g. Exodus 19,5; Deuteronomium 7,6; 14,2; 26,18). It is, therefore, possible to translate it as "singled out", "peculiar" or "precious", "dear," emphasizing the act of choice or the property of the object respectively. Mendelssohn favors in all cases "peculiar" and cognates even where the sense of "treasure" was favored by traditional translations and commentaries. King James' translation, Exodus 19,5 reads: "Now therefore, if ye will obey my voice indeed, and keep my covenant, then ye shall be a peculiar treasure unto me above all people: for all the earth is mine:" Mendelssohn translates: "So sollt ihr mein besonderes Eigenthum unter allen Nationen sein, denn mein ist die ganze Erde." However, the Arameic translation of Uncalos has "dearest of all people", and Rashi has "dear treasure ... that kings save."

It may be that Mendelssohn wished to tune down the connotation of "chosen people" as assigning a special value to the Jewish people and rather point to Israel's submissiveness to God.

sively to that genus, and the reason for this end must lie in the *differentia*, for if it were in something common to many genera, then this *segula* would also be present in the individuals of another genus. For example: due to the shortage of our understanding, the laughter in Man may be unrelated to the faculty of the intellect which constitutes the essence of man, and we therefore call it *segula*, but there is no doubt that He, may He be blessed, gave laughter to man to some end, and the reason for this end is therefore in the faculty of speech [i.e. the intellect], for if it were in life or sensibility it would have been common also to the beasts and to the birds. But although our intellect does not suffice to apprehend the constitution of the essence of [even] one of the beings such that nothing remains unknown to us, there is no doubt that each genus has one or many *segulot* that are connected to the *differentia* of the genus in a connection unknown to us, thus that we do not know why this genus has that *segula*.

The novelty in Mendelssohn's concept of *segula* is first that he explicitly concludes from the co-extensionality of the *segula* and the *differentia*, i.e. the essence, that there must be an intrinsic connection between them, second that we do not know this connection due to the limits of our (present?) understanding. It is thus not excluded and perhaps even assumed that a progress in our understanding may reveal such intrinsic connections between the essence and the *segula*.

Now, since we know that Maimon thought of the property "shortest" as a *segula* of the straight line[31], we can immediately translate Kant's elaborations on synthetic judgments a priori and on this specific judgment into Aristotelean-Leibnizian terms: A synthetic a priori judgment is a predication of a *segula* to a genus defined by the *differentia*. Further analysis either by our or by a more extended intellect may reveal how this *segula* is connected to the essence of the substance. If it turns out that the concept of the *segula* is implied by that of the *differentia*, the judgment would prove to be not synthetic but analytic. With this, Kant's project would be subverted and Leibniz's vindicated. Maimon explicitly draws this concu-

31. GM 75-76.

sions. In his essay on the progress of philosophy since Leibniz, Maimon demands that the "principle" of a science follow from the "*differentia specifica*" of its subject matter, for otherwise it lacks "the necessity and universality required by science." (GW IV, 65) In other words: The *segula* and all other properties of a substance should be implied by the definition.

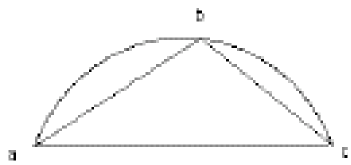
However, Maimon's discussion modifies Mendelssohn's deliberations in one essential point: it is not entirely clear whether Maimon thought that for an infinite understanding the *segula* would prove to be contained in the concept of the substance, and would thus be analytic. At least for our intellect, Maimon clearly wishes to differentiate between a *segula* which necessarily *follows* from the definition of the subject and a property which is *contained* in this definition, or, as he sometimes says, between a property which is (implicitly) contained in the definition of the object, and a property which follows from the object itself. The dilemma is that if all thought is analytic, then it is indeed clear why it is apodictic and universal; however, it would also prove empty, since all containment is a (partial) tautology. If it is synthetic, then we owe an explanation as to how it nevertheless can follow from the definition and, therefore, be universal and apodictic.

The discussion above showed that the phenomenon to which Kant referred with the term "synthetic judgment a priori" was by no means unknown. It was rather common philosophical knowledge in the Aristotelian tradition and named "*idion*," "*proprium*" or "*segula*." An innovation apparent in both Mendelssohn and Kant is that they find this phenomenon in need of explanation, and each of them offers explanation of this. In the spirit of Leibniz, Mendelssohn suggested that there must be a conceptual connection between the *proprium* and the esssence which we do not yet understand due to the limits of our understanding. Kant famously suggested that the additional property arises in the synthesis of understanding and intuition in experience and is not reducible to a conceptual connection.

2.2. *Maimon's Proof that the Straight Line is also the shortest between Two Points*

In his Transcendentalphilosophie Maimon attacked Kant's contention that the proposition "The straight line is also the shortest between two points" is a synthetic judgment a priori and involves intuition. Maimon attempted to show that this proposition is in fact analytic, i.e. that the property "shortest between two points" can be inferred form the definition of a straight line. He thus attempted to vindicate Mendelssohn's suggestion that the concept of the essence must somehow imply the *proprium*.

The proof proceeds from Christian Wolff's definition of a "straight line". Maimon remarks that no alternative has yet been offered to this definition. The definition states that the straight line is the line "the parts of which are similar to the whole".[32] Maimon interprets this as stating that all parts have the same direction. If we abstract from the magnitude, the parts of a line can be distinguished from each other only by their "direction" (*Richtung*) or their "position" (*Lage*). But if this is so, then a straight line (abstracted from its magnitude!) has no parts or is one line only, since it is defined by its singular direction. A not-straight line is in fact "several" lines individuated by the change of direction. This reduction of a perceptual quality ("straight") to quantity ("one", "several") contradicts of course, Kant's view that the "concept of the straight contains nothing of quantity, but only a quality" (*CpR*, B 16) Having established this equivalence between "one" and a "straight" line, Maimon ventures to prove that the predicate "shortest between two points" is implied by the subject term "A straight line". If successful, this proves that the allegedly synthetic proposition, "A straight line is the shortest between two points" is in fact analytic.



Suppose that between a and c there is one (i.e. straight) line and also the broken lines ab, bc. Euclid's *Elements* I, 20 proves that two sides of a triangle are longer than the third, and hence ab+bc › ac. Now, since any multilateral figure can be analyzed into triangles to which Euclid's I, 20 applies, it follows that "several" lines, i.e. all other lines between a and c

32. "... eine Linie deren Teile dem Ganzen ähnlich sind." See Wolff: Mathematisches Lexicon. Leipzig 1716 (WGW I, 11; S. 806): „Linea recta, eine gerade Linie, Ist, deren Theile der ganzen ähnlich sind." Vgl. hierzu Wolff: Elementa Matheseos Universae, Halle 1713. Tomus 1, Definitio 7 (WGW II, 29; S. 122) und Anfangsgründe aller mathematischen Wissenschaften, Halle 1710. Erster Teil, 4. Erklärung (WGW I, 12; S. 119). I adopted these references from Ehrensperger's annotations to his edition of *Transcendentalphilosophie.*

will be longer than the one [straight] line ac.[33]

Elements I, 20 proves that the *straight* line is shorter than two straight lines between the same two points. Curved lines are not considered in Euclid. Maimon's proof attempts to apply Euclid's proposition also to curved lines:

> ... a straight line means: *one* (according to its position) line, and a not-straight (curved) [line] means several lines (thought as one by their common rule). I hence wish to prove analytically this proposition: that *one* line is shorter than several [lines] between the same points. (Tr 68)

The proof proceeds, as Maimon says, *per substitutionem*: The curve between two points is substituted by a broken straight line, which can be resolved into triangles, to which *Elements* I, 20 applies (Tr 65-66, 68). The new and crucial move is hence the substitution of the broken for the curved line. The equivalence between a curved and a broken line was widely accepted, in fact it was part of the definition of the curved line. Christian Wolff, whose defintion of a straight line Maimon adopted, defined in a complementary way the curved line as the line "the parts of which are not similar to the whole line or can be well distinguished from it", and that is "compounded of infinitely small straight lines" or a "a many-sided polygon of infinite-

33. "Hr. Kant führt diesen Satz: eine gerade Linie ist die kürzeste zwischen zweien Punkten, als einen synthetischen Satz a priori z.B. an. Laßt uns aber sehen: Wolff definiert eine gerade Linie: eine Linie deren Teile dem Ganzen ähnlich sind (vermutlich, deren Teile einerlei Richtung haben; weil die Richtung das einzige ist, woran man eine Linie erkennen und von andern unterscheiden kann); und da Linien abstrahiert von aller Größe, nur durch ihre Lage verschieden sein können, so heißt eine gerade Linie so viel: als eine (der Lage nach) Linie, und eine nicht gerade (krumme) so viel als mehrere Linien (die durch ein ihnen gemeinschaftliches Gesetz, als eine einzige Linie gedacht werden). Ich will also versuchen, diesen Satz: daß nämlich eine Linie (zwischen zweien Punkten) kürzer sein muß als mehrere (zwischen denselben Punkten), analytisch zu beweisen. Ich setze also zwei Linien, die ich mit einer, zwischen denselben Punkten vergleichen will. Hieraus entspringt in der Anschauung ein Dreieck, wovon Euklides (Buch I. Satz 20.) bewiesen hat: daß die zwei Linien zusammen genommen (Seiten des Δ) größer sein müssen als die dritte, und dieses bloß durch einige Axiomen und Postulate, die aus dem Begriff analytisch folgen. Z.B. eine gerade Linie zu verlängern, die Lage der Figuren verändert in ihrer Größe nichts, u. dergl. Eben dieses kann auch vom Verhältnis dieser einen Linie mit mehrern, die mit ihr zwischen eben den Punkten enthalten sind, leicht bewiesen werden; weil immer eine geradlinige Figur die in Dreiecke aufgelöset werden kann; entstehen wird. Laßt uns setzen z.B. die Linie ac ist mit dreien Linien a d, d e, e c, zwischen eben den zweien Punkten a, c, enthalten. Ich sage also: die Linie ac muß kürzer als die drei Linien a d, d e, e c zusammengenommen sein. Denn aus vorigem Satze erhellet, daß $a c < a b + b c$. $b c = b e + e c$. folglich $a c < a b + b e + e c$: nun ist aber: $b e < b d + de$ folglich $a c < a b + b d + d e + e c$. Q. E. D. (Tr 65-67)

ly many and infinitely small sides."[34]

We now understand why Maimon does not provide a diagram. The proof is not and should not be geometrical and dependent on construction but conceptual. Maimon does not show by means of a diagram, as Archimedes did, that between a concave curve and a straight line that connect the same two points, we can describe a broken line which is longer than the straight and shorter than the curved line. Maimon argues conceptually. The curve and the broken line are equivalent; they are "several" lines (in the simple case: two), opposed to the "one" straight line. We may, therefore, substitute "several" straight segments for a "curved" line. Two points connected by one straight line and also by a broken line of two segments form a triangle. From *Elements I, 20* we know that two sides of a triangle are longer than the third, therefore the curved/broken line is longer than the straight line between two points. Note that "The straight line is the shortest between two points" is not overtly analytic since its negation is not an overt contradiction.[35]

Maimon hence proves that a straight line between two points is shorter than a curved line between the same points by substituting the broken for the curved line according to Wolff's definition and based on the proposition that triangles may be substituted for all multilateral figures ("weil immer eine geradlinige Figur die in Dreiecke aufgelöset werden kann" Tr, 66-67). *Elements* I, 20 is applied but not discussed by Maimon. In fact, Maimon believes that the proof of *Elements* I, 20 itself is also analytic, proceeding "by means of some axioms and postulates that follow analytically from the concept [of the straight line]" (Tr 66). Two years later than Maimon, Johann Christoph Schwab raised this very same claim concerning *Elements* I, 20 in the *Philosophisches Magazin*. He, too, wished to show that Kant's geometric example of a synthetic judgment a priori was in fact analytic but he did not discuss curved lines. Even his opponent, Rehberg, could not show that *Elements* I,20 was not analytic.[36]

34. See Wolff's definition of a "curve" quoted in #3.6 below.

35. Tr 358, note to Tr 26. Another possibility is that the proposition is a covert tautology as Maimon suggested later, when he wrote his "Short Synopsis". If I say that the straight line is not the shortest, "I contradict myself, since distance can only be determined by the shortest line", i.e. because we measure distance by straight lines (Tr, 177-178).

36. See J. Chr. Schwab, "Über die geometrischen Beweise, aus Gelegenheit einer Stelle in der A.L.Z., Philosophisches Magazin 3 (1791), 397-407. A.W. Rehberg, "Über die Natur der geometrischen Evidenz", Philosophisches Magazin 4 (1792), 447-460. See the discussion in J. Webb, "Immanuel

Again, here too, Maimon was on firm ground for his time.

It is however important to see what exactly Maimon's claim and contribution are. Maimon does not claim that all of geometry is analytic, on the contrary. Without intuition we would not even know what a "line" is. The relata of comparison (the lines) must be given in intuition. Only their relations, the identity and difference in number and magnitude, are pure concepts of the understanding.[37] The gist of Maimon's proof is the substitution of "one" for "straight", of "several" for "curved" such that *Elements* I, 20 may be applied not only to a broken line but also to a curve. This is his original contribution. This contribution is based on Wolff's definition of the "curve" as consisting of infinitely many straight segments. It was precisely this assumption on the part of Wolff which Maimon (later) criticized and rejected in his discussion of the circle (see below # 3).

Now, it seems that on the basis of the same presuppositions the very same proof as Maimon's was given both in the sixth century and at the beginning of the twentieth. However, it is the differences that put Maimon's precise intention in relief. In his commentary on the relevant definition of Archimedes, Eutocius maintains that a "curved" (or "bent") line in Archimedes refers to both a curved and a broken straight line and then proceeds to present a proof that the straight line is the shortest between two points. Eutocius connects two points AC (see figure 1 above) with a straight line and also with another line, "concave in the same direction" (i.e. that all its points are either on the straight line itself or on one side of it, none on the other side).[38] He then chooses an arbitrary point B on the concave line and inscribes

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Kant and the Greater Glory of Geometry", in: Naturalistic Epistemology, ed. by Abner Shimony and Debra Nails, Dordrecht etc. (Reidel) 1987, pp. 17-69, here 31-33.

37. "Freilich muß die Einheit oder Mehrheit der Linien (ihrer Lage nach) konstruieret, d.h. in einer Anschauung dargestellt werden, ohne welches diese gar keine Bedeutung hätten: aber das heißt nur: die Glieder der Vergleichung (die Gegenstände), nicht das Verhältnis selbst wird in einer Anschauung dargestellt. So wie wenn ich sage: das Rot in a ist mit dem Rot in b einerlei; so ist der Satz analytisch, obschon die Gegenstände der Vergleichung gegebene Anschauungen sind. Hier ist eben der Fall: eine gerade Linie ist so wie eine nicht gerade Linie (viele Linien unter einer Einheit gebracht) in einer Anschauung gegeben; aber nichts destoweniger ist das Verhältnis selbst (daß die erstere kürzer als die letztere ist) analytisch (durch den Satz der Identität und des Widerspruchs, per substitutionem) bewiesen." Tr 67- 68.

38. Eutocius remarks that Archimedes' definition of a curved (or "bent") line refers to "any line in a plane, without qualifification, which is other than straight; any single line in a plane, compounded in whatever way, so that even if it is composed of straight lines ..." (Netz, 2004, p. 244). Archimedes' definition is rather a postulate assuming the existence of such lines that "lie wholly

the triangle ABC. By *Elements* I, 20, it follows that ABC is longer than AB. The same procedure can be repeated for the segments AB and BC, and "doing this continuously, we shall find the closer straight lines to the ABC to be ever treater. So that it is evident from this that the [concave] line itself [ABC] is greater than AB."(Netz, 245-246).

Now, this is not at all Maimon's proof. Eutocius' proof is geometrical, not conceptual. It proves by means of a diagram, not by substitution of a definition for a concept. It thus proves that the concave line is longer than a broken line inscribed into it, not the equivalence of these lines. Finally, the proof inscribes ever more triangles into the concave curve and refers on the one hand to *Elements* I, 20 to conclude that two sides of a triangle are together longer than the third and, on the other hand, it tacitly refers to the diagram, i.e. to intuition, to establish that the concave curve in each case is longer than the two sides of the inscribed triangle. "Longer" being a transitive relaiton, it follows that if the concave curve is longer than the two sides of the inscribed triangle, and the two sides of the triangle are longer than the third, then the concave curve is longer than the straight line between two points. However the proof presupposes that the concave line is longer than the two sides of the triangle inscribed into it, and this is known by intuition only, not proven! It is exactly this proposition which Maimon sets out to prove.

The claim that curved and broken lines are equivalent or that the straight line is merely a special case of a curved line is specifically modern, as Wolff emphasized in his definition of a curve ("in der neueren Geometrie"). On the basis of this equivalence (further buttressed by the modern concept of "limit"), Louis Couturat produced in 1873 essentially the same proof as Maimon's and presented it again in 1904 in criticism of Kant. Almost in the same words as Maimon, Couturat, too, emphasized that he does not claim that geometry is independent of intuition, but rather that when basic definitions are accepted, the relevant proposition can be inferred without the aid of intuition.[39]

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on the same side of the straight lines joining their extremities;" see Archimedes, On the Sphere and the Cylinder, definition 1

39. Louis Couturat, "Kants Philosophie der Mathematik" (first published in: "Revue de Métaphysique et de Moral", May 1904), in: *Die philosophischen Prinzipien der Mathematik* (Leipzig: Dr Werner Klinkhardt) 1908, S.. 247-326, esp. 292-296.

In sum then, Maimon claimed to prove something Archimedes was content to formulate as a postulate, and Eutocius erroneously believed he could prove. Maimon and Couturat proceeded from the equivalence of a curved and a broken straight line and then proved that the straigt line is the shortest between two points. Both of them did not maintain that geometry as such is analytic.

If successful, Maimon's proof refutes Kant's claim that the proposition "The straight line is the shortest between two points" is synthetic, and since this was Kant's exemplification of a synthetic judgment a priori in geometry, Maimon would have jeopardized Kant's project and the proof would turn the proposition to be in favor of Leibniz! The proof would allow us to define "straight line" by "shortest": "A straight line is the shortest between two points". This substitution rids us of Euclid's obscure definition of the straight line and shows that the perceptual quality it attempts to capture is essentially the clear concept of the understanding "shortest", which merely appears to perception in an unclear way as "straight"[40].

Now, the problem with this proof is that it depends on the alleged equivalence of a curved line (eine nicht gerade (krumme)) with "several" straight lines (mehrere [gerade]) and thus substitutes one for the other. The legitimacy of this substitution is essential to the argument and Maimon will later in the book criticize precisely this alleged equivalence.[41]

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40. Certainly, in this case "shortest" resp. "distance" would have to be introduced.

41. Kant himself raised a different objection. He was not satisfied with Maimon's interpretation of Wolff's definition of the straight line. Maimon interpreted the "similarity" between all parts of the straight line as referring to the identity of "direction". Kant objected that "direction" already presupposes the notion of "straight", hence intuition. However, Kant did not suggest an alternative definition. See Kant's letter to Herz, 26.5.1789. Maimon attempted to answer Kant (Tr. 68-70), but later he used this point to criticize Kant: If the definition of direction and straight presuppose each other, how can we formulate a construction rule for the straight line? And if we cannot, what does it then mean to present a concept in intuition? In a later note, Maimon refers to a hypothetical objection against this proof. Since he introduced in a similar fashion also Kant's objections, it is not excluded that this objection was in fact raised by someone, however this was not Kant. The objection is that Maimon's proof presupposed that the not-straight line between two points can be considered as two sides of a triangle, the third side being the straight line between the same two points. Since a triangle is only possible if two of its sides are longer than the third, the proof was a petitio principii. Maimon answered, that he presupposed only that a triange could be constructed such that one of its sides is the line in question and then proved that the sum of the other two sides is indeed longer than the one initially given (Tr, 367-368).

2.3. *Kant's Critique and Maimon's Answer*

The manuscript of *Transcendentalphilosophie* was sent to Kant on April 7, 1789 accompanied by letters of Maimon and Marcus Herz.[42] At this time Kant was engaged in the controversy with the authors of the *Philosophisches Magazin*. Whereas Kant understood that Maimon, like the authors of the *Philosophisches Magazin*, above all Johann August Eberhard, criticized him from a so-called Leibnizian position, he also recognized the differences, at least in the quality of their objections.[43]

In his answer to Herz's and Maimon's letters, Kant addresses Maimon's proof that the straight line is also the shortest between two points and criticizes the (Wolffian) definition of the straight line which Maimon used since it contains a vicious circle. The straight line was defined with reference to its direction, but "direction" in its turn is defined with reference to "straight". Kant adds, however, that this is but a "bagatelle" ("*Kleinigkeit*").[44]

Kant did not criticize Maimon's proof nor his substitution of a plurality of straight lines for a curved line. However, he did extensively criticize the ultimate tenet of Maimon's proof, namely to refute the claim that knowledge depends on both understanding and intuition which are independent of each other and not reducible to one another.[45]

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42. See the letters by Marcus Herz and Salomon Maimon of April 7, 1789 to Kant in: AA XI, 14-15 and 15-17. GW VI, 423-425 and 426-427.

43. Kant's often-quoted praise for Maimon was not yet read on the background of this controversy although the connection is obvious. On April 9, 1789, Carl Leonhard Reinhold recommended Kant to publicly announce that Eberhard and others did not understand him ("daß man ... Sie nicht verstanden habe."). Kant answered on Mai 12 that "to say that Mr. Eberhard, as many others did not understand me, ist the least that can be said". On May 24 then he writes Herz that "none of my adversaries understood me and the main issue (Hauptfrage) as well as ... Mr. Maymon".

44. Kant to Marcus Herz, May 26, 1789, AA, XIpp. 53-54. Maimon attempted to answer Kant (Tr. 68-70), but more important here is a footnote in which he addresses his motivation. "My task here is only to show: that according to the definition of a straight line mentioned above the proposition: A straight line etc. [scil. is the shortest between two points] is not an axiom but a proposition that can be analytically inferred from others." (Tr 66-67, note) Given any other definition, says Maimon, he could prove as well that the proposition is not synthetic but analytic. Maimon is hence not committed to this or that definition (on the contrary, he says that he disagrees with Wolff and he criticizes the definition later) (Tr, 67, note and 68-70).

45. Kant emphasizes in his discussion two of Maimon's claims that were diametrically opposed to the groundwork of the *Critique of Pure Reason*:

In *Transcendentalphilosophie* we find Maimon's answers to Kant's criticism both in the body of the text and in a note, one of the few at the bottom and not at the end of the text. In the text itself, Maimon names Kant's objection to Wolff's definition of the straight line (Tr, 68), and almost verbatim quotes it (Tr, 70) but maintains that it is irrelevant to the import of his proof:

> "If Mr Kant wishes not to accept Wolff's definition of a straight line (since no other definition exists, as far as I know), but holds the straight line for a concept determined merely by intuition, then we have here an example of how the understanding can turn a concept of reflection into a rule of producing an object (which, in fact, should be thought between already existing objects, not first produce them by thinking this concept) (Tr, 68)."

It seems that Maimon suggests the following argument: The concept "shortest" falls under the concept of reflection "difference" (applied to magnitude). Being "shortest" between two points is not merely a property of the straight line, but the essential property of this line. "Straight" - a concept determined by the intuition, as Kant said - turns out to be merely a "picture" (Bild) or a distinguishing mark (Merkmal) of it. Indeed, when geometrical propositions depend on the properties of the straight line, it is the property of being the shortest between two lines, not straight. (Tr 69)[46]

- First, that intuition and understanding are not independent of each other (as Kant claimed) but that intuition is confused knowledge of what in thought is a clear concept.

- Second, that the concepts of the understanding are objectively valid because human understanding is of the same kind as "divine understanding", which is the "creator of the forms and of the possibility of things (in themselves) in the world".

Kant adds that according to Maimon our understanding is also "a part" (ein Theil) of this infinite understanding, and this is what Kant dubbed and criticized as Spinozism. Now, that the individual soul or the intellect is part of the "divine understanding" or of a "soul of the world" (anima mundi) is not in the printed text of the Transcendentalphilosophie, nor says Kant that it was to be found in the manuscript. Kant rather quotes Maimon's expression that the human intellect was "the same" as the infinite intellect and takes it to mean that it was part of the divine intellect. And, in fact, Maimon did entertain this view and maintained that it is necessary, among other things, to justify the claim to objective knowledge.

46. "Das Geradesein ist gleichsam ein Bild oder das Merkmal dieses Verhältnisbegriffes: daher kann es auch nicht als ein Verstandsbegriff um irgend eine Folge daraus zu ziehen, gebraucht werden.

Maimon hence reverses the relation of "essence" and "*proprium*" of the line in question. Whereas Christian Wolff defined this line as "straight" and predicated of it that it is also "shortest between two points", Maimon maintained that the definition of the line which refers to its essence, should be the concept of the understanding "shortest between two points", and that "straight" is merely a "distinguishing mark" (Merkmal) by which it is easily recognized in perception. This suggestion in itself is not incompatible with Kant's basic claims: a concept of the understanding (shortest) is presented in intuition and a new property (straight) is now known.

However, Maimon's proof seems to vindicate the Leibnizian program: a perceptual property was successfully reduced to a concept of the understanding. The property "shortest" is the "essence" of the line and this shows in geometry. "Straight" is merely a handy perceptual distinguishing mark of the "shortest" line. Thus, we also recognize in everyday practice human beings by their perceptual properties, e.g. their erect posture, but their essence is nevertheless *animal rationale*. Interpreted this way, Maimon's proof corroborated Leibniz's thesis that perception is but confused thought. Euclid's definition of the straight line was notoriously obscure, and so was also Wolff's. The proof that the straight line is the shortest between two points showed that this line could be defined by the clear concept "shortest", which merely appears in an unclear way as "straight" to perception. Kant's paradigmatic example was turned into that of Leibniz!

Maimon drives the point home in a footnote which he added in response to Kant's letter:

> "My intention here is merely to show: that according to the quoted definition of a straight line, the proposition: A straight line etc. [scil. is the shortest between two points] is not an axiom, but a proposition analyti-

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Man mag alle Sätze der geraden Linie durchgehen, so wird man finden, daß dieselben, nicht in so fern sie gerade, sondern bloß in so fern sie die kürzeste ist, daraus folgen; so wenig als von allen andern sinnlichen Anschauungen etwas anders folgen kann, als daß sie das sind, was sie sind." Tr 69

This move was repeated some decades later by the renown mathematician A. M. Legendre. See his *Elements de Geometrie*, douzieme edition, Paris (irmin Didot), I, livre 3, Definitions: " III. La ligne droite est le plus court chemin d'un point a un autre. IV. Toute ligne qui n'est ni droite ni composee de lignes droites est une ligne courbe." (p.1) (Heath, I, 169)

> cally inferred from others. And suppose that we nevertheless finally hit on synthetic propositions on which all others are based (I leave undecided as yet whether this is the case), I nevertheless maintain that just as by means of my definition I rendered analytic this proposition which was claimed to be synthetic, I can do the same with these [synthetic propositions] too." (Tr, 66-67, note)

Maimon hence believed not only that his proof was successful in this case, but also that he discovered a general procedure to render geometrical propositions (perhaps with the exception of the axioms) analytic. We start out with a definition of the object in perceptual terms (here: "straight"), we analyze this concept and substitute for the perceptual property a concept of the understanding (here: "one line"), and then prove that the seemingly synthetic judgment follows analytically from the axioms of geometry and logic. The allegedly synthetic nature of geometry is due to its dependence on intuition - and this is due to the finiteness of our understandings - just as Mendelssohn (following Wolff, following Leibniz) said.

Maimon evidently believes that the burden of proof in this controversy is Kant's. He does not attempt to prove that mathematics is analytic, but rather expects Kant to prove that it is synthetic and is content to refute Kant's arguments. This would be different if Maimon were to claim that geometry is entirely analytic. The *onus probandi* is not equally distributed, it rests with the proponent of the new thesis, and the opponent, representing the received view, needs but refute the new claims raised. One problem remains, however. Can we construct the concept of a straight line in intuition? This is an essential element in the arguments of both Kant and Maimon. In order to answer this question, we have to briefly discuss the notion of construction.

2.4. *Definition, Construction, Proof in Euclid and Kant*

Euclid's geometry is said to construct its objects with a ruler and a compass. This is misleading since it obliterates an essential difference between Euclid's constructions and the early modern notion of construction shared by Kant. In Euclid, construction produces objects from simpler elements i.e. the straight line (a rectilinear segment) and the circle, it does not produce the straight line and the circle themselves. Using "ruler" and "compass" instead of "straight line" and "circle" insinuates that the latter are to be drawn; this is not so. The straight line and the circle are introduced as *given* by postulates 1-3 of book I of *Elements*. When Euclid postulates "To draw a straight line from any point to any point" (postulate 1) or "To produce a finite straight line continuously in a straight line," (postulate 2) or, finally, "To describe a circle with any centre and distance" (postulate 3), he does not draw the lines or the circle, nor, therefore, the points on them. He simply works with the primitive objects themselves, with the rectilinear segment and with the circle. These primitive entities are postulated as already constructed, as given. It is, therefore, misleading to speak of "Euclidean" construction with ruler and compass. Beginning with a "point" and constructing a (straight) line or a circle by imagining the motion of this point, hence using an imaginary ruler and an imaginary compass, has been known since antiquity but marginalized because it was deemed to lack rigor and because generation of mathematical objects was conceived to be incompatible with their eternal nature. This method of construction became prominent and integrated into the mainstream in early modern mathematics, and its notion of construction was projected on construction in Greek geometry.[47] Kant, too, shares this early modern understanding of con-

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47. This has been extensively and convincingly argued by David R. Lachterman in his *The Ethics of Geometry*, chapters 1 and 2, pp. 1-123:

"All the constrcutions in the *Elements* are in fact performable using only ruler and compass; and yet, not only do Euclid and his commentator Proclus say nothing about this, but on the evidence of the text further restrictions on the allowable use of these simplest instruments were part of Euclidean strategy." (71)

Lachterman observes that in the case of real, more complex constructions "With only a few notable exceptions, Euclid chooses to put these verbs (for 'operation' in construction - G.F.) in the perfect passive imperative. Bisecting a line-segment at a point is expressed as 'let it have been cut in two' (...); describing a square on a line is 'let it have been described on AB'; 'contriving' (Heath's idiom) that A is to B as C is to D is 'let it have come about that' (...). The importance of this stylistic trait is twofold: First, Euclid does not give instructions or permission to a reader to carry out a specified operation but castes the operation into impersonal, passive form; second, the perfect tense tells us that the relevant operation has already been exectued prior to the reader's encounter with the unfolding proof (of a theorem or of a problem, the use of the perfect is uniform in these two classes of propositions.)" (65)

The age of the Scientific Revolution explicitly formulates the alternative program, namely to construct also the straight line and the circle. In Newton this program is both formulated and declared to be impossible within geometry: "... the description of right lines and circles, upon which geometry is founded belongs to mechanics. Gemoetry does not teach us to draw these lines, but requires them to be drawn ... Therefore geometry is founded in mechanical practice ..." (Preface to the first edition of the *Principia*, Cajori xii). Kant quotes this locus in the preface to his *Metaphysical Foundations of Natural Science* 1786 (xxiv). See the discussion in Webb, Kant and Geometry, 22-27.

struction. However, the demarcation between those who banned motion from geometry, and those who allowed it, was not merely historical: it also ran between more and less philosophically stringent conceptions of geometry and proof. Constructing objects by motion or proving theorems by congruence threatens to replace logic and understanding with the evidence of the eyes or imagination. Kant's warrantor in mathematics, Johann Schulz, banned motion from geometry, Abraham Gotthelf Kästner, a major mathematical authority (also for Kant), allowed it.[48]

The first, obvious and important difference between Kant's and Euclid's programs is hence that Euclid constructs all geometrical entities in the plane from straight lines and circles, he does not construct the straight line and the circle themselves. But this is at the core of Kant's program: We cannot "think" a line unless we draw it in pure intuition and synthesize the successive segments as one line. (*CpR* A 100-103) The same holds for the circle as one curved line. The primitive elements from which Kant proceeds are hence but one: a point. The (a priori, non empirical) motion of a point in the imagination (pure intuition) produces a line, straight or curved.

> "However, that the possibility of a straight line and a circle can be proved, not mediately through proofs, but only immediately, through the construction of these concepts (which is not, to be sure, empirical), stems from the fact that among all constructions ... some must be the

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Contemporary scholars as well as Kant's contemporaries usually focus on the difference between empirical and a priori construction in pure intuition and do not pay attention to the difference to which Lachterman draws our attention. The difference is moreover obliterated in German (of the eighteenth century): here both a compass and a circle are called "Zirkel" (Cirkel). G.S.A. Mellin e.g. writes in his article "construieren": "Die mechanische Construction ist diejenige. welche durch andere Werkzeuge, als Cirkel und Linie gemacht wird;" (*Encyclopädisches Wörterbuch der kritischen Philosophie*, 1. Bd., 2. Abtheil, Züllichau und Leipzig (bei Friedrich Frommann) 1798, S. 835.) However, on the next page he speaks twice of "Cirkel und Lineal". It seems that he does not distinguish between these pairs.

48. See below (# 3.3) on Kästner's definition of the circle. Schulz writes in his *Anfangsfründe der reinen Mathesis*, Königsberg 1790, that if mathematics is to grant "insight" (Einsicht) and not be restricted to a mere mechanism "then the demonstrations have to be followed in all strictness as far as possible. The eye has no say here." (p. iii-iv)

first."[49]

It is only by presenting to pure intuition a moving point that we can "think" a straight line; it is by turning a thus produced rectilinear segment around one of its ends that we produce a circle. This immediately prompts a number of questions: Is "motion" a priori or a posteriori (in which case geometry would not be a priori)? Does "motion" presuppose invariance under translation, e.g. of the radius revolving around one end and thus constructing a circle? Does motion produce a "continuous" line as required by prostulate 2? Moreover, Kant famously said that to "construct a concept" means to exhibit (darstellen) a priori the intuition "corresponding" to it. (*CpR*, A713/B741). What does this mean? How do we accomplish this task and mediate between the concept and intuition? And how do we know that what we constructed in intuition indeed "corresponds" to the concept?[50]

The answers to these questions lie in the specificity of mathematics. According to Kant, it is only in mathematics that we have precise concepts. This is so because only in mathematics (in contradistinction to empirical knowledge) the definition does not explicate the concept of a given object, but rather defines an object a priori and constructs it (in accordance with the defintion). But how do we know that the object is really "adequate" to the concept or "corresponds" to it? Kant does not explicitly discuss the question, but he seems to presuppose that the construction rule is identical to the definition or implied by it.[51] If this is so, then indeed the constructed object must be adequate to the definition. However, if the construction rule is not implied by the definition, we must prove that the object has the properties named in the definition. In the latter case, the "construction of a concept in intuition" would involve three steps instead of two: the defintion, the construction, and the proof that the object con-

49. Kant, "Über Kästner's Abhandlungen,", AA xx, 411, transl. by Lachterman, op. cit. p. 53

50. Certainly, there is a vast literature on the role of "construction" in Euclid and Kant's philosophy of mathematics, but there is rarely a discussion of the questions raised above.

51. See the discussion of the definition and construction of the circle below. Mellin says that the intuition corresponds to the concept if all the determinations (Merkmale) of the concept can be perceived in in the diagram. "Diese Anschauung correspondirt dem Begriff, heisst, es sind in ihr die ... Merkmale anzuschauen, die in dem Begriff gedacht wurden." (article "Construiren", p. 814.) In line with the distinction between "nominal" and "real" definition (without using the terms), Mellin suggests that in mathematics only "real" definitions are proper definitions whereas Euclid's definitions e.g. are merely nominal. See loc. cit., p. 831.

structed satisfies the definition. In this case there is no difference between an object we constructed (not according to its definition) and an empirically given object: in both cases we have to prove that the object conforms to the concept.[52] We will see below that the difference between definition and rule of construction is essential to Maimon's criticism of Kant.

2.5. *The Construction of the Straight Line*

What is the construction rule for the "straight" line? Is it implied by its definition? Or how do we prove that the line we draw by the motion of a point is indeed "straight"?

Kant nowhere names a rule of construction for the straight line, nor does Maimon. Kant does say that we draw a line by the motion of a point in pure intuition but not by what rule we guarantee that this line is "straight". In the context of his proof that the straight line is also the shortest between two points, Maimon once claimed that the concept of reflection "shortest" was also serve as "the rule of producing" this line (Tr, 68). However, he did not specify how we are to construct the line according to this rule.

In his letter to Maimon, Kant named a difficulty in the definition of the straight line. Although Kant himself did not realize it, this difficulty is the reason why there is no rule of construction for the straight line. Suppose that we were to construct a straight line in pure intuition by the motion of a point, as Kant suggests. Suppose further that in the construction of a line in intuition we can distinguish between the understanding and pure intuition: the understanding effects the "transcendental synthesis of the imagination" by which we think the line successively produced as one line (*CpR* A101-102). Suppose further that the motion involved in describing the line and thus space itself is an a priori concept (*CpR*, B 153-155). Nevertheless, we still lack a rule of construction for the straight line as distinguished from a curved or a broken line. The Wolffian definition of the straight line does not provide such a rule. Kant correctly argued that "straight" presupposes "direction" and "direction" "straight" (AA XI, 53-54). This definition of the straight line is thus circular. "Straight" is known in intuition but there is no rule for its construction. In the notes at the end of *Transcendentalphilosophie*

52. Kant once referred in passing to this problem. "Thus the empirical concept of a plate is homogeneous with the pure geometrical concept of a circle. The roundness which is thought in the latter can be intuited in the former." *CpR* A137/B176 However, Kant does not specify what "roundness" is, but I take it that it refers to the standard definition of the circle.

which were certainly written later than the main body of the book and only after Maimon received Kant's letter, Maimon clearly recognized the problem:

> "What is the pure construction of a straight line supposed to be as we have no definition of it and hence cannot name a rule for its generation a priori?" (Tr, 368)

Without an adequate definition, there cannot be a rule of construction or a criterion by which we would judge that the object constructed "corresponds" to the concept. The property "straight" is recognized as such in intuition and that is all (Tr, 368)[53] There is therefore no difference between the construction of a straight line and the judgment that a line given in intuition is straight. If we construct a geometrical object according to a rule, we have insight into its generation, or so-to-say into the "necessity of [its] possibility" (Tr, 171).[54] This is not the case with the straight line that we cannot construct according to a rule.

We thus see that Maimon radically changed his mind. He disagreed with Kant both before and after this change. However, in the manuscript sent to Kant, he presented a proof that rendered the proposition "The straight line is also the shortest between two points" analytic. Later he acknowledged that "The straight line is the shortest between two points" is synthetic, and therefore evidently recognized that his proof was wrong. Moreover, in response to Kant's letter both in the second chapter of *Transcendentalphilosophie* and in the "Short Synopsis" at the end of the book as well as in the notes to this text, Maimon maintained that since we have no adequate definition of the straight line, nor, therefore, a rule for its construction, "straight" is merely a perceptual property that cannot be reduced to a concept of the understanding. However, since this is so, Kant's suggestion to construct the line by the motion of point was not helpful: there is no "rule" of such construction, hence also no "genetic" defini-

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53. This is also Hume's opinion: "We may apply the same reasoning to CURVE and RIGHT lines. Nothing is more apparent to the senses, than the distinction betwixt a curve and a right line; nor are there any ideas we more easily form than the ideas of these objects. But however easily we may form these ideas, 'tis impossible to produce any definition of them, which will fix the precise boundaries betwixt them. When we draw lines upon paper, or any continu'd surface, there is a certain order, by which the lines run along from one point to another, that they may produce the entire impression of a curve or right line; but this order is perfectly unknown, and nothing is observ'd but the united appearance." Treatise I,iv. Nidditch, p. 49.

54. See below (# 2.6) for the similar view of Kästner.

tion of the straight line. It turns out that, pace Kant, "straight" is merely recognized in intuition just as color is (CpR, B 743). Now, since Maimon's proof failed, he had to admit that the proposition "the straight line is the shortest between two points" is synthetic. However since there was no rule of construction for the concept of the straight line, also Kant's interpretation of this proposition failed.

Kant maintained that synthetic judgments a priori are possible because we construct a concept in intuition. However, it has been shown that we do not construct the concept of the straight line in intuition, since we have no such concept (definition) or rule of construction but merely recognize straight lines in intuition. The alterantives are therefore that "The straight line is the shortest between two points" is either analytic or synthetic; and if the latter that the possibility of this judgment and of synthetic judgments a priori in general is as yet a mystery.

2.6. *The Turn to Empircial Skepticism (and Rational Dogmatism)*

At the end of the Transcendentalphilosophie, in his "Short Overview of the Entire Book", which was certainly written after receiving Kant's letter[55], Maimon again discussed the proposition that the straight line is the shortest between two points. The conclusion of Maimon's renewed discussion was entirely different not only from Kant's view (that this proposition is synthetic a priori, necessary and universal) but also from his own former one. Maimon previous discussion was intended to show that the proposition was a priori as Kant said, however, pace Kant, not synthetic but analytic. In his present discussion, Maimon concludes that (for us humans!) the proposition is dependent on empirical intuition and therefore synthetic, however not a priori but empirical. He maintains that Kant's argument that we have synthetic a priori knowledge because we construct concepts in intuition, fails because there is no rule of construction for the straight line. This, of course, also flatly contradicts his assertion discussed above (# 2.5; Tr, 68) that "shortest"" provides a rule of construction for this line. Hence no a priori application of a concept of the understanding to intuition takes place. Since there is no such rule, empirical intuition alone guides the construction of the straight line, and

55. Engstler, p. 30, note 12 notes that Maimon answers here Kant's allegation concerning Spinozism in his letter to Herz and Maimon. This proves that the "Short Overview" was written later than the body of the manuscript.

intuition alone also shows that it is the shortest between two points. This is the diametrically opposed position to the view that the proposition in question can be reduced to an analytical proposition which Maimon advocated in his first discussion. It is reminiscent of Hume's discussion of the same question in his *Treatise of Human Nature*. Hume wrote:

> "Nothing is more apparent to the senses, than the distinction betwixt a curve and a right line; nor are there any ideas we more easily form than the ideas of these objects. But however easily we may form these ideas, 'tis impossible to produce any definition of them, which will fix the precise boundaries betwixt them. When we draw lines upon paper, or any continued surface, there is a certain order, by which the lines run along from one point to another, that they may produce the entire impression of a curve or right line; but this order is perfectly unknown, and nothing is observ'd but the united appearance. ... 'Tis true, mathematicians pretend they give an exact definition of a right line, when they say, it is the shortest way betwixt two points. But in the first place I observe, that this is more properly the discovery of one of the properties of a right line, than a just deflation of it. For I ask any one, if upon mention of a right line he thinks not immediately on such a particular appearance, and if 'tis not by accident only that he considers this property?
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> Treatise of Human Nature, Book I, Part II, chapt. III.4, § 3., Nidditch, 49-50[56]

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56. There is reason to believe that Maimon knew Hume's Treatise even before his change of mind. In the locus where he presented his early opinion, namely that "straight" can be reduced to "shortest", he seems to answer Hume's argument that "straight" is an internal, "shortest" an external property. Maimon wrote: Es scheint ein Paradoxon zu seyn, da man gemeiniglich glauben mögte, hier sey das Geradeseyn eine innere Bestimmung (Verhältniß der Theile unter einander) und die kürzeste seye eine äußere Bestimmung. Bei genauer Überlegung aber findet sich gerade das Gegentheil: nämlich daß das Geradeseyn oder die Einerleiheit der Richtung der Theile, die Entstehung derselben schon voraussetzt. Daher taugt auch diese Definition der geraden Linie nichts. Die Wolfische Erklärung kann dieser Schwierigkeit nicht ausweichen; weil die Ähnlichkeit der Theile mit dem Ganzen bloß in der Richtung seyn muß, folglich setzt es schon Linien voraus. Die Eigenschaft aber, daß sie die kürzeste sei, fängt gleich mit der Entstehung an, und ist zugleich ein inneres Verhältniß. Tr 70. Compare this with Hume, Treatise, I,ii.4 (p. 50): "A right line can be comprehended alone; but his definition (shortest, GF) is unintelligible

Maimon now argues that if the straight line is known in perception only, then also the proposition that it is the shortest between two points must be founded in perception and is merely empirical and a posteriori, an empirical generalization from repeated perceptions:

> "It may be that I judge that a straight line is the shortest between two points because I always perceived it to be the case and it therefore became for me subjectively a necessity etc. This proposition hence has high probability, but no objective necessity (Tr, 173)[57]

Later (See below in this chapter), Maimon will revise this view and admit that the proposition "The straight line is the shortest between two points" is independent of repeated experience and known on the first time a straight line is seen.

Since there is no doubt that we are compelled to affirm that the straight line is the shortest between two points, and since we fail to understand the reason for this truth, we should strictly distinguish between (logical) necessity and intuitive compulsion. Compulsion is a feeling and not an objective criterion for a priori and therefore universally valid truths.[58]

Now, this would be different if we could speak with Kant of the objects of geometry as constructions of concepts and thus form a bridge between the understanding (concept) and in-

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without comparison with other lines, which we conceive to be more extended."

57. "... daß ich z.b. urtheile, eine grade Linie ist die kürzeste zwischen zwei Punkten, kann daher rühren, weil ich es immer so wahrgenommen habe, daher ist es bei mir subjektiv zur Nothwendigkeit geworden." (Tr, 173)

58. „Der Dogmatiker geht aus seiner innern Wahrnehmung aus, daß z.B. die gerade Linie die kürzeste zwischen zweien Punkten ist. Dieses hat als Faktum seine völlige Richtigkeit. Wenn er aber behauptet, daß jedes denkende Wesen überhaupt, unter allen Umständen, die gerade Linie als die kürzeste zwischen zweien Punkten denken müsse, so sagt er mehr, als er, seiner innern Ueberzeugung nach, mit gutem Gewissen sagen kann. Denn da er die nothwendige Verknüpfung zwischen dieser besondern Anschauung (gerade Linie und dieser Form (die kürzeste) nicht einsieht, so bleibt immer der Zweifel übrig: ob nicht ein anderes denkendes Wesen, oder auch er selbst unter gewissen Umständen diese Anschauung mit einer andern Form verknüpft denken könne.

Der Skeptiker bleibt daher in den Gränzen seiner innern Ueberzeugung stehn, und wagt nicht, einen Schritt weiter zu thun. [...] Zwischen dem humischen und dem kantischen Skeptizismus ist der Unterschied gar nicht so groß, als man uns bereden will. Jener nimmt Fakta für Fakta an, und zieht die Nothwendigkeit und Allgemeingültigkeit dieser Fakta in Zweifel. Dieser setzt zu Gunsten der wissenschaftlichen Form, diese Nothwendigkeit und Allgemeingültigkeit, die an sich möglich ist, voraus." (GW III, 245 f.)

tuition (object). The object could then be claimed to be an instantiation of the concept. However, we do not have such rules of construction (Tr, 423, note to Tr 173; see the discussion below). Since we cannot bridge understanding and intuition, our conceptual and perceptual knowledge is separated into apodictic a priori logical knowledge on the one hand, and uncertain a posteriori perceptual knowledge on the other.

Judgements are objectively true that depend either on the principle of contradiction or on the categories of the understanding.[59] The condition of being a judgement is to conform to the principle of contradiction; the condition of being an object is to conform to the categories. A judgment that applies to an object merely by virtue of being an object, therefore applies universally to all objects as such (*Gegenstand überhaupt*; Tr 168)[60]

From this argument it follows that a priori in the narrow and absolute sense means that the judgment precedes the representation of any specific object (Tr, 172), and from this it immediately follows that geometrical axioms are not strictly a priori, as they are about specifically determined objects, not about any object whatever. Certainly, geometrical axioms are relatively a priori in that they precede knowledge of specific objects and are about the most general properties of all material objects, i.e. about their spatial properties. Therefore, the axioms are materialiter a priori (dependent on the properties of the most general objects in question - space and time - and independent of the properties of specific objects), but they are not purely formaliter a priori (dependent only on the form of judgment and therefore valid of all objects whatever), but involve the specific human forms of intuition.[61]

Consider the following example: the proposition "A straight line is not straight" is a pri-

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59. "Objektive Notwendigkeit kann nur dem Satze des Widerspruchs (in so fern es eine nothwendige Beziehung eines Subjekts überhaupt auf ein Objekt überhaupt bedeutet), oder den Kategorien (in so fern dadurch in Beziehung auf unser Subjekt ein reelles Objekt überhaupt gedacht werden kann), nicht aber einem sich auf ein besonderes Objekt bezeihenden Satze beigelegt werden." Tr 423, Note to Tr 175.

60. According to Maimon, propositions true of an object as such belong to logic. "Die Logik ist die Wissenchaft des Denkens eines durch innere Merkmale unbestimmten und bloß durch das Verhältnis zur Denkbarkeit bestimmten Objekts überhaupt" *Versuch einer neuen Logik*, §1, GW V, 1 (vgl. § 3, V,4). In Kantian garb, Maimon speaks of the unity of general and transcendental logic; in Aristotelean garb we could speak of the "principles" which are - in our terms - both logical and ontological and therefore pertain to being as being. See Aristotle, Metaphysics IV, 3.

61. On "materialiter" and "formaliter" see also Tr, 51-52.

ori false in the absolute sense since we need not know the specific object (a straight line) to know that "not straight" contradicts "straight" and that the judgment above cannot be true (Tr, 169). Such strictly a priori judgments are hence those that are true of "an object as such" (Tr, 168). The proposition "The straight line is the shortest between two points," however, depends on what a straight line is and not on its being "an object as such." This proposition, therefore, cannot be strictly a priori (Tr, 171-172).

But why believe it was a priori in the first place? Kant, says Maimon, adduced this example and claimed that it was a priori because it "expresses necessity, and must therefore be a priori." (Tr, 173). Now, Maimon does not question the claim that the proposition in question "expresses necessity". He does however question that this necessity is "objective". It may be subjective "compulsion". The difference is fundamental: "objectively" necessary (or true) is only what is logically true, i.e. dependent on the understanding alone. This is so because there is but one form of the understanding embodied in logic (general and transcendental). Compulsive or not, whatever is dependent also on sensibility, could be merely "subjectively" true. There may be other thinking beings who have other forms of sensibility than our forms of space and time and what to us is compulsive may not be so at all A "subjective" truth may of course be also "objectively" true, but we cannot know this for sure. In both cases we would sense "compulsion" but would not know whether this is because a proposition follows from the forms of the understanding (and is objectively true) or from the forms of sensibility (and is merely subjectively true).

Maimon hence takes another line of argument. It does not proceed from "necessity" to "truth", but the other way around: from the difference between logic and intuition to the difference between objective and subjective truth. Both forms of truth appear in one and the same form of compulsion. Kant's mistake was to believe that he could reason also the other way around: from "compulsion" to (objective) "necessity", and from "necessity" to truth.[62]

62. See especially Tr 174-175, 151f, but also "Ueber Wahrheit", GW I, 599-616. Maimon is consistent in this view. "Objectively necessary" are only propositions governed by the law of no contradiction and the categories alone. Kauferstein believes that Maimon upheld two opposed conceptions: that mathematical propositions are objectively resp. subjectively true. He quotes only one locus to support the claim that Maimon thought mathematical propositions are objectively true. There, Maimon first elaborates the distinction between objective and subjective truth as above, and then says: "The mathematical propositions are hence objectively true, but only under the condition that their axioms are objective (since this is surely possible)." (GW I, 602).

In his "Short Overview" at the end of *Transcendentalphilosophie* Maimon suggested that the proposition, "The straight line etc." may be, as Hume believed, an empirical proposition of very high probability (based on repeated experience) but of no objective necessity (Tr, 173)[63] Later, however, Maimon notices an important difference between this and other empirical propositions. We judge that fire causes the melting of wax because we repeatedly observe that it melts when we place it near fire. Not so the straight line: We do not learn from repeated experience that the straight line is shortest. Rather, We cannot imagine the straight line even once without imagining it shortest. This is so on the first as well as on subsequent occasions. Put differently: fire and wax may appear independently of each other, not so the straight line, for the straight line cannot be constructed without being shortest and vice versa. In the first case we have independent objects, in the latter an object and its *proprium*.[64] In the

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Maimon's formulation is clumsy, and yet unambiguous. If and only if the axioms are objectively true and the derivation proceeds according to logic (which is by definition objectively true), will the derived propositions be objectively true. The fact that the (geometrical) axioms are imposed on us in intuition and we therefore have no insight into the reasons of their truth does not mean that they are not nevertheless objectively true. We may know them by intuition (in a "confused" manner) due of our finiteness, whereas they are objectively necessary in themselves. This is certainly possible, but we cannot know whether it is a fact or not. Moreover, we know that Maimon in fact believed that the Euclidean axioms are "metaphysically" true, but the emphasis is on "believed", he did not "know" it. All we can say is that the propositions of geometry are imposed on us in intuition and that they can be applied to the world of experience, hence that they are "real", but not that they are objectively true. (GW I, 602f) Maimon's position is here consistent. The tension is not between opposite views of his but between what we "believe" and what we "know". (See Kauferstein, 209 - 220. Kauferstein seems to mistake "objective" and "subjective" truth and vice versa. He writes e.g. that metaphysical and logical truth are subjective (240). The opposite is of course true: "Objektive Notwendigkeit kann nur dem Satze des Widerspruchs (...), oder den Kategorien (...), ... beigelegt werden." Tr 423, Note to Tr 175.

63. "... daß ich z.B. urtheile, eine grade Linie ist die kürzeste zwischen zwei Punkten, kann daher rühren, weil ich es immer so wahrgenommen habe, daher ist es bei mir subjektiv zur Nothwendigkeit geworden." (Tr, 173)

64. Fragt man mich ferner: warum halte ich den Satz z.B. Eine gerade Linie ist die kürzeste für absolut; diesen aber: das Feuer schmelzt das Wachs, für bloss comparativ nothwendig und allgemeingültig, da ich doch für beide kein anderes Kriterium habe, als dass ich es mir immer nicht anders vorgestellt habe? so erwiedere ich hierauf: Es ist ein grosser Unterschied zwischen diesen beiden Sätzen. Dass eine gerade Linie die kürzeste ist, habe ich mir immer als einen Satz, vorgestellt. Dahingegen dass das Feuer das Wachs schmelze, habe ich mir erst nicht als einen Satz, sondern bloss als eine zufällige Begebenheit (dass auf die Gegenwart des Feuers das Wachs geschmolzen ist) vorgestellt; und nur durch eine verschiedene Mahl wiederholte Beobachtung dieser zufälligen Begebenheit, machte es mir die Gewohnheit (die selbst nach empyrischen Gesetzen wirkt) gleichsam zur Nothwendigkeit, das Schmelzen des Wachses als nothwendige Folge von der Gegenwart des Feuers vorzustellen." (IV, 372-373)

first case we have an empirical generalization, in the second a synthetic judgment a priori. And yet, although we know that the line is necessarily the shortest between two points when we construct it, we do not know why this is the case; even if our productive imagination (*Erdichtungsvermögen*) constructs the line according to a rule, we do not *understand* the rule. (Tr, 19-20; 59) In contradistinction to Kant, Maimon considers then synthetic judgments a priori not as a solution of a problem but rather as a problem itself. Maimon hence concedes Kant that there are three kinds of proposition: analytic and necessary, synthetic and contingent, and finally synthetic and yet universally true. The latter are judgments on "*propria*", or synthetic judgments a priori.

It may seem as if Maimon did not take into consideration the difference between empirical and pure intuition. This is not the case. However, Maimon stresses the difference between recognizing what is *given* in pure intuition and what is *constructed* in pure intuition. The straight line is not constructed in pure intuition according to a rule of the understanding. If we construct a geometrical object according to a rule, we have insight into its generation, or so-to-say into the "necessity of [its] possibility" (Tr, 171)[65]. This is not the case with the

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"Ich denke die grade Linie nothwendig als die kürzeste, ich mag sie zum erstenmal vorstellen, oder ihre Vorstellung schon oft widerholt haben." (IV, 215) Maimon continues with a comparison to the proposition that fire melts wax.

This is reminiscent of Hume's distinction between the mathematical and the empirical sciences (Enquiries Concerning Human Understanding IV,1 (#20)). In the mathematical sciences a single instance suffices to establish knowledge, in the empirical sciences a "number of similar instances" are required. The assertions of the mathematical sciences based on a single instance are certain, those of the empirical sciences based on a number of instances are merely probable. Maimon revised his attempt to interpret the proposition that the straight line is an empirical proposition exactly because he discovered that it is a certain judgment based on a single instance. The distinction above between "zufällige Begenbenheit" and "Satz" presumably refers to Kant's distinction in the Prolegomena, # 20 between "Wahrnehmung" and "Erfahrung". Perception becomes experience when it is subsumed under a concept of the understanding in a judgment. According to Maimon, there are no "assertory judgments"; what normally passes for an assertory judgment is a mere perception (Wahrnehmung) (GW V, 411). I suppose that Maimon refers with "Satz" zu "Urteil". Kant there also gives the example of the judgment that the straight line is shortest between two points.

65. Similar views were expressed short time later by Abraham Gotthelf Kästner in the Philosophishes Magazin: "14) Was also Euklid als möglich beweist, ist nothwendig möglich; In einem Verstande, der alle Kentnisse der Eelementargeometrie faßt, ist der Begriff davon; Er kann seyn, wäre zu wenig gesagt. Und weil sich die ganze Geometrie eigentlich imVerstande des Geometern befindet, so heißt bey ihr wirklich seyn, im Verstande seyn; folglich ist bey ihr alles Mögliche wirklich." 16) Möglich in der Bedeutung: Kann seyn, kann auch nicht seyn, kennt die Geometrie

straight line [66] The fact that we have no rule of construction for the straight line shows that Kant explanation that these judgments depend on the construction of concepts in intuition is wrong. It remains an open question whether they are virtually analytic, merely subjectively necessary, or governed by another principle.

Having given up the hope of being able to prove that the proposition "The straight line is the shortest between two points" is objectively valid, Maimon wished to show that if it is true for a given straight line, it is true for all straight lines irrespective of their magnitude.[67] The argument, which is presented rather poorly, may be rendered in the following way: A truth of a judgment is not dependent on its extension. It is merely important that it be applied to its proper objects. To err in the application of the judgment "The straight line is the shortest between two points" would be possible only if there were more than one kind of straight lines and the judgment - true of one kind only - were to be applied to another. However, the class of straight lines cannot have sub-classes to which the judgment does not apply. A straight line can have no further determinations but its magnitude. Therefore the question may be concretely put thus: Does the property "shortest between two points" depend on the length of the straight line? To exclude this possibility, Maimon produces a proof. He shows that if the judgment is valid for the distance ab, then it is also valid for the distance ac = 2ab, and if it is valid for ac, then it is also valid for ab = 1/2 ac. (Tr, 176-177). Hence the judgment "The straight line is the shortest between two points" is independent of the absolute length of the straight line and therefore cannot be applied improperly. Since it is true, it is true for all straight lines.

2.7. *Synthetic a priori and proprium*

In the "Short Overview of the Entire Work" Maimon no longer attempts to prove the judg-

nicht." (Abraham Gotthelf Kästner, "Was heißt in Euklids Geometrie möglich? erläutert von A.G. Kästner, Philosophisches Magazin, hrgg. von J. A. Eberhard, Bd. 2, St. 4, Halle (Johann Jacob Gebauer) 1790, 391-402, here: 400 and 401.

66. Note to Tr 423, note to Tr 98.

67. Maimon addresses here the problem why proofs which use a singular representation are valid for all instantiations of the represented genereal concept. Kant answers this problem in his distinction between a particular image (Bild) and a universal schema (*CpR* A140-142/B179-181). See also Friedman, 41, 90.

ment "The straight line is the shortest between two points". Here he merely asserts that like all other true propositions this one too must have "a ground in the object" (Tr, 175), and this objective ground means that "for an infinite intellect the proposition must be analytic, but we have no insight into [this ground]." (Tr, 181) The failure to prove this proposition does not entail that it is not analytic in itself. It may be that only an infinite intellect understands how the *proprium* is implied by the concept of the subject. The failure to prove the proposition may have been the fault of the (Wolffian) definition of the straight line from which the attempted proof proceeded. Whether we can prove the proposition or not, if it is true, and much more so: if it is necessarily true, the predicate must be somehow implied by the subject term, because we have no other notion of truth. The same holds here. If the proposition is necessarily true, then it is (virtually) analytic, because this is the only kind of truth we understand.

Maimon does not believe now that he must produce an adequate definition of "straight line" and show that it contains the property "shortest between two points" in order to maintain that for the infinite intellect the proposition must be analytic; "it suffices", he says, "that I believe it is not impossible." (Tr, 179). The onus probandi lies with the opponent who must show that the proposition is necessarily true and yet synthetic. Kant failed exactly to fulfill this task. But on other occasions, Maimon maintains that an analytic implication is not a thought at all. The question is whether between the Scylla of Empirical Skepicism and the Charybdis of tautology there is the narrow straight of "real synthesis", of a synthetic a priori judgment?

Maimon draws the conclusions from his discussion so far in his commentary on Baumgarten's Metaphyisca ("Meine Ontologie") which he appended to the *Transcendentalphilosophie*. He there distinguishes between a reason ("Grund") of the truth of a proposition and the condition of a "real object" (#14, Tr. 241-242). The relation of reason to consequence is that of the general to the particular judgment and is asymmetrical; that of the condition to the conditioned is that of a singular to a singular judgment and is therefore symmetrical and convertible. The example for the former is: "Every object is identical to itself" implies "Object A is identical to itself," but not vice versa. The example of the convertible condition-conditioned is: "The judgment that a line is straight (condition) and the judgment that it is shortest (conditioned), and also vice versa! In a "Concluding Remark" to his last book (Kritische Untersuchungen, 1797), Maimon introduces this fact as the criterion of synthetic judgments a priori and praises it as an "important discovery":

All primary geometrical propositions can be converted unchanged, since all original geometrical propositions are synthetic propositions." (GW VII, 362)

Now, exactly the convertibility of the proposition without change of quantifier was used by Ibn Tibbon in his explication of "*segula*" (*proprium*). in his *Dictionary of Foreign Terms* traditionally appended to his translation of the *Guide*:

> It is manifest that the *proprium* is equal [in extension] to the genus of which it is a *proprium*, i.e. that it extends neither more nor less than the genus, and may be exchanged with it in the proposition, i.e. that the *proprium* may be predicated of the genus or the genus predicated of the *proprium* - as he said: every laughing is a man and every man is laughing."[68]

The example Maimon gives for primary synthetic geometrical properties is "The straight line is the shortest between two points." (GW VII, 364; see also VII, 133ff) This co-extensionality of "straight" and "shortest" (line) cannot be explained. Maimon's "important discovery" concerning synthetic judgments a priori is simply that Kant did not improve on Aristotle! Both Kantian and Aristotelian formulations give no insight into the reasons of this co-extensionality; they merely give it a name. In his commentary on the *Guide* he writes:

> "And therefore although this judgment is true and although we cannot draw a straight line without drawing it shortest, this is so for the human understanding only - not for any understanding in general - since the predicate is not included in the subject. And therefore it is possible to imagine that a different[69] understanding exists for which the concept of a straight line is not conjoined with the condition that it is the shortest between two given points. And therefore the basis of these judgments is the law of synthesis in general (eine Synthesis überhaupt) i.e. that for

68. Shmuel Ibn Tibbon, *Bi'ur hamilot hazarot*, *segula*. In: Moses Maimonides, *Guide of the Perplexed*, Hebrew Version of Shmuel Ibn Tibbon, edited by Yehuda Ibn Shmuel, Jerusalem (Mossad Harav Kuck) (1947) 2000, p. 23- 24

69. Reading with Bergmann and Rothenstreich "acher" instead of "echad".

the concept of a determined object (zum Begriff eines bestimmten Objektes) it must obtain that there be some connection of its predicate with the subject in general. This is so since, if there is no such connection, it is impossible that the understanding will apply the predicate to its subject and that some necessary consequence will follow from it (und daß er daraus eine notwendige Folge ziehen kann). But [it is] not [necessary] that a specific subject will be connected with a specific predicate. And such judgments are called synthetic judgments (synthetische Sätze).

And therefore analytic judgments do not at all fall under the definition of belief since they cannot be conceived differently. And therefore the statement is not correct "I believe that the triangle is a determined extension", because this concept is not peculiar to me but to every conceiver endowed with understanding in general. But the synthetic judgments fall under the definition of true belief. For example: "I believe that the straight line is the shortest between two given points", because I believe that this concept that I conceive is not peculiar to me but to every conceiver as such, i.e. that there would be a contradiction here if the straight line were not the possibly shortest line, although I do not conceive where this contradiction is. Nevertheless, a contradiction is necessarily present, for otherwise this judgment would not be necessary for every conceiving subject - and understand this![70]

The duality is clear. There are "common notions" of two kinds: "analytical axioms" following

70. *Givat Hammore*, Commentary on chapter I, 50, p. 75-6, cf. 33-34. Maimon uses for the property in question the term "*segula*". Maimon characerizes such properties as "following necessarily" from the definition although not included in it. His example is that Man is a social animal which necessarily follows from his definition as a "speaking animal". (*Givat Hammore*, 21). In the present locus Maimon also gives the example of the triangle which is defined as a closed figure enclosed by three straight lines, from which definition it necessarily follows that it has three angles. Short time afterwards, this example will exemplify "subjective necessity" imposed on us by intuition without insight. See my discussion below, #4.2.

On the concept of "belief" see Kant *CpR*, B848-859.

from the definition of the concepts involved (e.g. "The whole is greater than its part" follows from the definition of "whole" and "part")[71]. These we understand. There are also "synthetic axioms" which follow not from the definition but from the construction of the concept ("aus der Konstruktion des Begriffes"). These we do not understand. The example Maimon gives is, of course, that the straight line is the shortest between two points: "For when we want to construct the concept of the straight line in our imagination (konstruieren in der Einbildungskraft) we shall necessarily find it the shortest between two given points."[72] The proposition is true, it may even be necessarily true, but we do not understand why.

The distinction between analytic and synthetic judgments is hence for human conceivers only. Human finitude means that human cognition depends also on intuition, on confused concepts. Two possibilities open up. The first is that the proposition depends on the specifically human sensibility. In this case the synthesis between "straight line" and "shortest" is not necessary and a different understanding could indeed deny that the straight line is also the shortest between two points. In this case, our belief in the proposition is peculiar to our sensibility or even the result of our habits. Kant attempted to explain how knowledge may be dependent on the form of sensibility peculiar to humans and yet necessary. This explanation failed because we do not in fact construct concepts in intuition. If, therefore, knowledge depends on sensibility, then it must depend on experience. Humans may be animals as all others who learn from repeated experience. "Rational" may mean not more than having specific habits and "truth" may be either an empty word or God's exclusive privilege. Human "understanding" may relate to truth not as the finite to the infinite, but as animal instinct to thought. This is Maimon's Empirical Skepticism.

The second possibility is that the judgment is indeed true for all thinking subjects. This will be the case if it is necessarily true and this means that its negation is a contradiction. Accepting that there are judgments that are necessarily true without insight into the reason of their truth, is tantamount to forsaking the hope of being able to elaborate Dogmatic Rational-

71. The fifth axiom of *Elements*, book I, reads: "The whole is greater thant the part." Clavius addedd: "The whole is the equal to the sum of the parts." This can also be taken as a definition of "whole" and "part", as Maimon seems to construe it.

72. *Givat Hammore*, Commentary to chapter I, 51, p. 77. The German in brackets is in the Hebrew original of the commentary, but written in Hebrew letters.

ism to completion and yet uphold the notion of truth underlying its program. This is Maimon's Rational Dogmatism (for the finite understanding). Maimon once imputed this position to Kant:

> I believe that Kant must have assumed the reality of synthetic judgments in respect to our limited understanding only; and I'll easily accord with him in this. (Tr, 62)

Kant with his "synthetic judgments a priori" and Maimonides with "*segula*" merely state the problem, the way in which the finite intellect conceives truth. But in both these forms truth remains opaque to reason. Philosophical analysis must reveal truth as it is in itself, i.e. for the infinite intellect i.e. as conceptually necessary. The alleged synthesis between Kant and Spinoza was hence tantamount to proceeding by philosophical analysis from Kant to Spinoza (or Leibniz), from the knowledge of the finite intellect towards truth (of the infinite intellect). On the other hand, the failure of this program speaks for skepticism and thus for Hume. In both cases, Kant's and Maimonides' positions are only a point of departure, not tenable philosophical positions.

2.8. *Maimon's Law of Determinability, Straight and Curved Lines*

The discussion above showed that we cannot be certain whether the proposition "The straight line is the shortest between two points" is necessarily true. The attempt to prove that it is analytic failed. This does not mean that no philosophical work can be done. If a predication is neither overtly analytic and necessary nor entirely arbitrary, but a real synthesis, then there must also be a "law of synthesis in general i.e. ... some connection of the predicate with the subject in general." (GM 75-76) This is where Maimon's Law of Determinability (Gesetz der Bestimbarkeit) comes in.

Maimon's Law of Determinability is a principle of concept formation from which also the theory of judgment follows. The principle states that every true synthesis of an "absolute concept" (so called to distinguish these subject-predicate syntheses from correlative terms) consists of a subject that can be thought independently of the predicate, and of a predicate that cannot be thought without the subject (Tr, 84). The "true" synthesis is distinguished from a merely verbal or empirical synthesis in that it implies new consequences which follow neither from the concept of the subject nor from the predicate taken separately. Moreover, The

Law of Determinability states that every predicate relates exactly to one subject and that only one predicate can refer to this subject.[73] It need not be discussed here whether Maimon believed that his theory of concept formation may also serve as an *ars inveniendi* or was content with the fact that it organizes knowledge in a Porphyrian tree of sorts. It suffices that it at least excludes ill-formed "syntheses" (such as "sweet line") and grants partial insight into the truth of real syntheses. It is not a criterion of true judgments (as a proof is), but a criterion of well-formed judgments that may be true.

Maimon did not apply his Law of Determinability to the straight and shortest line. This is understandable since the debated question was precisely whether "straight" and "curved" are alternative determinations of "line" or whether "straight" is but a limit of "curved". However, if this traditional classification of lines beginning with "straight" and "curved" branching off from "line" (see e.g. Aristotle, *Metaphysics*, 986a 25; *Anal. Post. 73b 19)*[74] is accepted, then we can see that "The straight line is the shortest between two points" is well-formed.

Maimon tells us that when he edited the chapter on "Subject and Predicate. The Determinable and the Determination" (chapter four of the printed book), he came across Gottfried Ploucquet's *Methodus calculandi in logicis inventa* (1763) and found in it an idea he also entertained, namely "that a judgment contains only one concept", i.e. explicates the subject term. Maimon copied four full pages (in the Latin original) from Ploucquet's into his *Transcendentalphilosophie* (381-384). Of special interest for our concern is that Ploucquet's example is the concept of the circle: "Every circle is some curved line". Now, the converse of the judgment is: "Some curved line is a circle" and the circle is distinguished from other curved lines by further determinations. Ploucquet suggests essentially the Euclidean definition: "A closed curved line within which a point is given equidistant from the points of the periphery". Read with an eye on his Law of Determinability, we obtain the following hierarchy: The first determination of "line" is "curved" (distinguishing it from: "straight"), the second is "closed" (distinguishing it from all "open" curves which do not enclose a figure), the

73. Tr, 86-87; 379 and Tr, II, 143. On the Law of Determinability see Oded Schechter, "The Logic of Speculative Philosophy and skepticism in Maimon's Philosophy. in: G. Freudenthal (ed.), (ed.): Salomon Maimon: Rational Dogmatist, Empirical Skeptic. Dordrecht (Kluwer) 2003, pp. 18-53.

74. On various classifications of lines, see Heath I, 160-165.

third is the specific "curvature" (here expressed by the condition that all points on the periphery be equidistant from the center) distinguishing between a circle and all other closed curves, e.g. an ellipse. (Tr, 382-383).

In reading Ploucquet, Maimon must have realized that if he accepted the traditional disjunction between "straight" and "curved" - and therefore abandoned Wolff's definition of the straight line that obliterated this distinction - then the Law of Determinability showed that Maimon's early proof that the straight line is also the shortest between two points is not merely wrong but a categorical mistake! This is so because "straight" and "curved" branched off from "line" before further determinations of "curved" produced "circle", and that, as concepts (as distinguished from intuitions), "straight" and "curved" are entirely disjunct. Since Maimon's proof was based on substituting several straight lines for a curved line, it was based on a categorical mistake.

But what convinced Maimon that the traditional classification of lines was preferable to Wolff's definition of the curved line as a plurality of straight segments? It must remain undecided what role Ploucquet's example played, but Maimon's own analysis of the construction of the circle was certainly decisive in this discovery. This will be discussed below. However, once discovered, the Law of Determinability showed that on logical (transcendental) grounds Maimon's proof that the straight line was the shortest between two points was built on sand. Maimon will spell out this insight in his discussion of the construction of the circle (see below # 3.6).

2.9. *Results: The straight line*

Maimon's repeated discussion in *Transcendentalphilosophie* of the proposition that the straight line is also the shortest between two points ends with a paradoxical result. If the proposition is objectively true, it is necessarily so, and this seems to imply that it is analytic. However, for all we know, the proposition - whether true or false - may be merely a highly probable empirical generalization. Because we are finite, we know neither whether the proposition is objectively true, nor, if so, how the predicate is entailed by the concept of the straight line. Put differently, if the notion of truth is meaningful then sensibility ("intuition") must be based on reason and must be a (confused) manifestation of it. But it may also be the case that Man is not an *animal rationale*, but merely an *animal sensuale*, guided by instincts and habits. With the exception of logic (and metaphysics) so-called Truth may be but an illusion. Both options are possible and both are supported by good arguments. There can be no resolution of the duality and Maimon therefore characterizes his philosophical position as Rational Dogmatism and Empirical Skepticism (Tr, 438-439). Who subscribes to this position? "As to the present, I know of none but myself", he says (Tr, 437).

However, Maimon's new and ambivalent position refers to human cognition only, not to propositions as such. Maimon did not change his notion of "Truth": If the proposition "The straight line is the shortest between two points" is true, then it is necessarily true. There is no other ground for truth than the entailment of the predicate in the subject concept, whether our analysis reaches thus far or not. Our knowledge reaches highest in geometry i.e. in the application not only of logic but also of the Law of Determinability. The latter enables us to exclude categorical mistakes and to recognize well-formed propositions, but it cannot single out the true propositions from all well-formed propositions. It is only "because of the shortage of our knowledge that such properties are synthetic. (Tr 178). Overt analytical propositions (AB ⊃ B; AB ⊃ A) are empty (GW V, 86-88), geometrically proven synthetic judgments involve intuition and cannot (conceptually, i.e. adequately) justify their claim to objective truth.

We see here how the gap between Maimon's concept of knowledge and his view of the best available human knowledge (mathematics) opens up. Maimon adheres to his notion that true knowledge requires insight (of the understanding), and this is possible only when the proposition is inferred from accepted premises. But he realizes that he failed to show that the test case of the straight line that is also shortest between two points can be reduced to this form. What failed in mathematics would fail a fortiori in other areas. Maimon draws radical conclusions: Human knowledge may in fact consist merely of empirical generalizations, but he nevertheless upholds the definition of truth in analytic terms. Since he cannot demonstrate that empirical knowledge can be reduced to a form proper to his notion of truth, he is content to state that his claim is at least not falsified:

> "Since all knowledge a priori must be analytic and it must be possible to infer it from the Law of Contradiction, how should we make analytic such statements that are synthetic due to the shortage of our knowledge? or how should we define the subject, such that the predicate be identical with it? ... I do not wish to take it upon myself, to develop all

> statements of this kind, in order to satisfy my demand; it suffices that I take it not to be impossible (Tr, 178-179). [75]

Here Maimon's mature conception of the duality of Dogmatic Rationalism and Empirical Skepticism is first elaborated[76]

Two consequences could be established concerning Maimon's critique of Kant. First, there is no rule of construction for the straight line and therefore Kant's explanation that synthetic judgments a priori are possible thanks to the construction of concepts in intuition fails here. Second, Maimon's attempted proof of this proposition failed too. once he recognized that a curved line cannot be equated with a plurality of straight lines. Hence, he had to admit that this proposition is synthetic (to the finite human mind at present), but he also refuted Kant's argument that the proposition is necessary because the property does not follow from the definition either conceptually or by a rule of construction. The property "shortest between two points" thus retained its status as a "*proprium*" of the straight line (or the other way around). It is co-extensive with the essence of the straight line but not part of it. Perhaps it could be proven by other means to be contained in the subject-term and the judgment rendered a necessary truth (rational dogmatism), or it could prove to be merely an empirical generalization (empirical skepticism). Kant dubbed such propositions "synthetic judgments a priori" but did not provide insight into their nature and did not improve on Aristotle's "*idion*."

We have not insight into the reason of the connection we make between two "straight" and "shortest", but we are nevertheless convinced that it is a fact, because it so appears in intuition:

> "The straight line is the shortest between [two] points is the apodictic conceived concurrence between two rules that the understanding prescribes itself in order to construct a certain line: (being straight and

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75. "Da alle Erkenntnis a priori analytisch sein muß, und sich aus dem Satz des Widersrpruchs herleiten lassen muß, wie sollen wir solche Sätze, die wegen Mangel unserer Erkenntnis synthetisch sind, analytisch machen? oder wie sollen wir das Subjekt definieren, daß das Prädikat mit ihm identisch sein soll? ... Ich will es nicht über mich nehmen, alle dergleichen Sätze auf diese Art selbst zu entwickeln, um dieser meiner Forderung ein Genüge zu leisten; genug, daß ich es nicht für unmöglich halte." (Tr, 178-179)

76. See Tr 436-437.

> shortest). We have no insight into why these two must concur in one subject; it suffices, that we have insight into the possibility of their concurrence (in so far both are a priori) ... in intuition. (Tr, 54-55)

Maimon's proof that the straight line is shortest between two points failed because he could not justify the substitution of "broken straight line" for "curved". He could have realized this when he discussed his Law of Determinability, he could also have realized this when he studied the relation between definition and construction in the case of the circle, which was Kant's other example of the construction of a concept in intuition.. Maimon recognized that here, too, the definition does not imply a rule of construction. However, we do have a rule of construction for the circle. This "genetic definition" of a circle involves motion and it has to be proven that its product satisfies the properties named in the definition of the circle used in geometrical proofs. The duality between definition and construction reflects the general duality of the understanding and intuition. Moreover, it showed Maimon specifically that the substitution of the broken straight line for the curve was unwarranted, hence that his proof that the straight line is the shortest between two points was untenable.

3. *The Circle*

3.1. *The Nominal and Real Definition of the Circle*

Problems concerning the circle, its concept and construction are central to *Transcendentalphilosophie.* This is so first because the circle (together with the straight line) is one of the two elementary objects from which all others are constructed in Euclid's geometry. This is so, second, because here the duality of concept and object, understanding and intuition shows most clearly. In his review of his of own book, written 1793, Maimon used the example of the circle to exemplify his views on the central topics of this book: the relation of the understanding to intuition, antinomies, appearance and the "thing in itself," the finite and the infinite intellect and more.

The duality of definition and construction in the case of the circle is already present in Euclid: We find in *Elements* a definition of the circle and then also a postulate securing that it can be drawn.

The definition is:

"A circle is a plane figure contained by one line such that all the straight lines falling upon it from one point among those lying within the figure are equal to one another." (*Elements* I, definition 15, Heath I, 153)

And the postulate:

"To describe a circle with any centre and distance." (*Elements* I, postulate 3, Heath I, 199)

This postulate is not a rule of construction. It states that a circle is a real object and thus avoids a problem which troubled philosophers in the seventeenth and eighteenth centuries. In geometry, objects can be defined which are logically possible (free from contradiction) but cannot be realized in space. Maimon's favorite example (adopted from Leibniz) for such entities was the decahedron, the perfect solid of ten equal faces.[77] Euclid's *Elements* end with the proof that there are exactly five such perfect solids, and the decahedron is not among them (Book XIII, props. 13-18). The decahedron is hence impssibile although its concept involves no contradiction. Kant's favorite example is the biangle, a closed plane figure of two sides only. (*CpR* B65; A220/B268)[78]

For some philosophers, the interest in the "real definition" of concepts reached further than the guarantee of the objects' possibility. It was also supposed to give insight into the essence of the object and its relations to its properties. This interest was most clearly formulated by Spinoza. Spinoza goes even further. He wishes not only to explain how such judgments are possible but also how they may be discovered. The proper way of discovery (recta inveniendi via) proceeds from the proper definition of the entity and infers its properties:

To be called perfect, a definition will have to explain the inmost essence of the thing, and to take care not to use certain *propria* [!] in its place. ... If a circle, for example, is defined as a figure in which the

77. See Leibniz *Nouveaux Essais* III, 15, § 3. See Maimon e.g. IV, 419 and *Logik* passim. Kauferstein erroneously believes that this is an "innovation" in respect to the rationalist theory (Kauferstein, 140).

78. The impossibility of the biangle is asserted in a presumably later addition to *Elements* I,4. It was eventually included in editions of *Elements* as an additional axiom. See Heath I, 232.

> lines drawn from the center to the circumference are equal, no one fails to see that such a definition does not at all explain the essence of the circle but only one of its properties. And though, as I have said, this does not matter much concerning figures and other beings of reason, it matters a great deal concerning Physical and real beings, because the properties of things are not known so long as their essences are not known.
>
> (*Treatise on the Emendation of the Intellect*, #94, 95; Curley, p. 39; translation slightly altered)

Spinoza then enumerates the essential requirements of a perfect definition:

> 1. If the thing is created, the definition, as we have said, will have to include the proximate cause. E.g., according to this law, a circle would have to be defined as follows: it is the figure that is described by any line of which one end is fixed and the other movable. This definition clearly includes the proximate cause.
>
> 2. We require a concept, or definition, of the thing such that when it is considered alone, without any others conjoined, all the thing's properties can be deduced from it (as may be seen in this definition of the circle). For from it we clearly infer that all the lines drawn from the center to the circumference are equal. (ibid., # 96; Curley, 39-40)[79]

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79. See Hobbes, *De Corpore* I,i,5, and his *Examinatio et emendatio mathematicae bodiernae*, second dialogue. On this conception of definition and its dependence on Hobbes (including the example given) see Ernst Cassirer, *Das Erkenntnisproblem* II, 90, 98ff; See also Cassirer's *Leibniz System*, 110-117. Leibniz emphasizes the importance of the constructive definition (in contradistinction to a nominal definition) to ensure the "real possibility" of the entity. See GP I, 384-385. See espeically "On Universal Synthesis and analysis, or the Art of discovery and Judgment", Loemker 229-233 (GP VII, 292-298). Leibniz uses there a property of a circle (proven in Euclid III, 20) to argue that there are "paradoxical properties" (GP VII, 291, Loemker, 280) of which we at first do not know whether they are real or not.

In fact, Spinoza's philosophical motivation here is not alien to Kant. On other occasions, he too considered this possibility of a "philosophical" definition which implies the properties. See his definition of a circle as "a line in the plane to which all lines drawn from one point are perpendicular." (AA 14, 23 drawing on Euclid's *Elements* III, 18 and implying proposition III, 35. For yet another such definition see AA 14, 31.See also Kant's Letter to Carl Leonhard Reinhold, As far as mathematics is concerned, this program was not peculiar to the rationalists.[80] Empiricists, too, upheld it in mathematics, but significantly denied it of "substances" of empirical knowledge. Locke writes:

> Thus a figure including a space between three lines, is the real, as well as the nominal Essence of a Triangle; it being not only the abstract Idea to whch the general Name is annexed, but the very Essentia, or Being of the thing it self, that Foundation from which all its Properties floww, and to which they are all inseparably annexed.
>
> (*An Essay Concerning Humane Understanding* III, 4, # 18; Nidditch, p. 418.

This is different in the case of substances, e.g. gold. Here we do not know

> "the real constitution of its sensible Parts, on which depend all those properties of Colour, Weight, Fusibility, Fixedness, etc. which are to be found in it.
>
> (ibid., # 19, p. 419)

The core of the rationalist program consists in denying that this distinction is essential and working towards supplying also for substances essential definitions which imply their properties. In his response to Locke, Leibniz argues that "real definitions" serve in the first place the distinction between possible and impossible entities. His example is the impossible Decahedron (*Nouveaux Essays* III. iii. # 15) which was adopted by Maimon from the *Nouveaux Essays* as he says.[81] But Leibniz also argues that the distinction between mathematical entities and substances and between properties and substances is not essential: we have essential definitions of some real substances, and we lack definitions of some properties (e.g. yellow, bitter). The difference between substances and properties lies in the degree of complexity and is not absolute. The aim of the rationalist program is to provide a definition of gold

May 19, 1789, AA XII, p. 43.

80. For Leibniz's discussion of "nominal" and "real" definition see, "Discours de Métaphysique", # 24, GP IV, 449-450 and "Meditationes de Cognitione, Veritate et Ideis", GP IV, 424-426.

81. GW VII, *Kritische Untersuchungen*, Dedication to Graf Kalkreuth.

from which all its properties (color, specific weight etc.) follow as all properties of the triangle (allegedly) follow from its (real) definition.[82]

Now, in the case at hand the question is first whether the constructive definition of the circle proves its existence and whether its properties follow from this definition. Since the properties of the circle are those either named in the definition or proved on its basis, the question above is tantamount to the question whether the proposed constructive definition is implied by the definition commonly used in geometry or implies it. Or again: Do we know, and how do we know, that the figure constructed by turning a segment around one of its ends satisfies the definition of a "circle" as the figure enclosed by one line all points of which are equidistant from a point within it.

3.2. *Kant on the Definition and Construction of the Circle*

The disagreement between Kant and Maimon over the circle revolves first around the relation of the circle's definition to the rule of its construction. In the case of the straight line, the extant definitions were obscure and no rule of construction was suggested by Kant (or anybody else). This is different in the case of the circle. Euclid gave a satisfactory definition and Kant adopted a rule of construction common at the time. However, whereas Kant believed that the definition of a circle implied its rule of construction, Maimon maintained that they are entirely different, in fact heterogeneous: the definition of a circle is a concept of the understanding, its rule of construction belongs to intuition. There is no point in which and no way by which intuition and understanding can meet. A synthesis of intuition and understanding is an empty word.

Not so Kant. In the "Discipline of Pure Reason" of the *CpR*, Kant maintains that a definition should contain clear, sufficient, and not more characteristics than necessary to refer to exactly the concept intended. These requirements, so Kant believes, can be met only in mathematics.

"For the object which it thinks it exhibits a priori in intuition, and this

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82. Maimon adopts this program:"Da wir aber in der Erfahrung sie einmal (in Zeit und Raum) verbunden antreffen, so setzen wir voraus, daß das (uns unbekannte) Wesen des Goldes so beschaffen seyn muß, daß die gelbe Farbe mit den andern Merkmalen desselben in gedachtem Verhältnisse gedacht werden müssen.-" (GW VI, 210)

> object certainly cannot contain either more or less than the concept, since it is through the definition that the concept of the object is given." (B 757-8).

How, then, should the circle be constructed according to the definition of the circle?

Kant defines the circle more or less as Euclid did :

> A circle is a line all points of which are equidistant from a single one (the center). (*CpR* A, 731-2/B 759-760)

Kant does not discuss here the rule of construction, but he does so in another context in the *Critique of Pure Reason* and assigns it the traditional term "postulate". He says:

> Now in mathematics a postulate means the practical proposition which contains nothing save the synthesis through which we first give ourselves an object and generate its concept -- for instance, with a given line, to describe a circle on a plane from a given point. Such a proposition cannot be proved, since the procedure which it demands is exactly that through which we first generate the concept of such a figure. (*CpR* A234/B287)[83]

To describe a circle "with a given line" obviously means to rotate a segment around one of its ends. And this is exactly what Kant says in letters to Reinhold and a week later to Herz and Maimon. The construction rule, so Kant says there, is

83. "Nun heißt ein Postulat in der Mathematik der praktische Satz, der nichts als die Synthesis enthält, wodurch wir einen Gegenstand uns zuerst geben und dessen Begriff erzeugen, z. B. mit einer gegebenen Linie aus einem gegebenen Punkt auf einer Ebene einen Cirkel zu beschreiben; und ein dergleichen Satz kann darum nicht bewiesen werden, weil das Verfahren, was er fordert, gerade das ist, wodurch wir den Begriff von einer solchen Figur zuerst erzeugen." (*CpR*, B 287) The term "practical" should not be here understood as refering to practically drawing a circle on paper. Kant erroneously imputed Maimon this misunderstanding (AA XI, 53) to which Maimon replied in a note, in which he stated that the construction in intuition suffices to render the concept possible, i.e. real (GW II, 42). In a different context, Maimon repeated that the construction in intuition gives the concept "objective reality" (objective Realität) and that it therefore lacks nothing to being real (wirklich). Drawing the circle on paper "does not belong to the concept of a circle and cannot be used in any real synthesis with it ... The circle is hence real (wirklich) already through its possibility." (GW IV, 649-650)

"turning a straight line around a fixed point."[84]

And Kant there maintains also (without an argument) that this rule follows from the definition or is implied by it:

> The "possibility [of a circle] is ... given in the definition of the circle, in that it is actually constructed by means of the definition itself."[85]

It is now also clear why Kant maintains that "such a proposition cannot be proved." The proof should show that the constructed object is adequate to the definition. But if the definition is itself the rule of construction, the adequacy required is immediately given. The proof could only repeat that the objects is constructed according to its construction rule, i.e. according to the definition. We place one end of the radius at one point (the center) and its other end either describes the circumference of the circle (with all its points) or coincides with already existing points and shows that their distance from the center equals the length of the radius. The proof coincides with the definition and with the rule of construction and is indeed redundant (presupposing, of course, that the radius is invariant under motion).[86]

The method adopted by Kant to construct the circle is not only a way to construct one

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84. Letter to Marcus Herz, May 26, 1789; Correspondence, p. 315, cf. 306; AA Vol. XI, p. 53. See also the letter to Reinhold of May 19, 1789, AA XI, 43.

85. Letter to Marcus Herz, May 26, 1789; *Correspondence*, p. 315, cf. 306, AA Vol. XI, p. 53. Lazarus Bendavid, whose book *Versuch einer logischen Auseinandersetzung des Mathematischen Unendlichen* [...]. Berlin 1789, was discussed by Maimon in *Transcendentalphilosophie* (pp. 275, 291sq) takes the very same position. On construction in general he says: „Der sicherste Probestein für die logische Richtigkeit mathematischer Begriffe ist, wie wir gesehen haben, die Möglichkeit ihrer Konstruktion. Eine Sache, die nun beim Unendlichen gar nicht angehet. [...] Denn bei ihnen ist die Konstruktion eben dasjenige, wodurch sie evident werden, aber auch Beweis für ihre innere Möglichkeit. Beim Unendlichen aber bestehet seine innere Möglichkeit eben darinn, daß er nicht konstruirt werden kann." (p. xxxi). And on the circle: "Bei der Konstruktion hingegen ist der konstruite [sic] Begriff mit der Konstruktion einerlei. Sie stellt das Allgemeine in Besondern vor." (p. 94) "Die Konstruktion des Begriffes Zirkel, ist wahre Konstruktion, indem der Begriff Zirkel eben das aussagt, was der vorgezeichente [sic] Kreis anschaulich macht." (p. 95)

86. See *Prolegomena*, # 38; AA IV, 320-321. Of course, we could also use the construction rule as a definition and thus ensure that definition and rule of construction coincide. A. G. Kästner begins his "Geometrie" (1764) with twenty definitions. All of them have the structure "x is", "x is called" (ist/sind; heißt/heissen). There is only one exception: the circle (def. 13): "Ein Kreis (circulus) entstehet, wenn sich die Linie ... um den festen Punkt ... herumdrehet." (*Anfangsgründe* I, 183) However, in this case we have to introduce the property that all points on the circumference are equidistant from the center.

of the two elementary objects of geometry. It is the paradigmatic example of a "real definition" which does not only explain the essence of the object (its constitutive property) and shows how it is produced, but also exhibits how its properties are implied by its essence. If successful, such a definition would render "synthesis" intelligible, meaning by this the connection of the essence to its "*propria*," which are not included in the concept of the object.

3.3. *Maimon's Critique of Kant's Construction of a Circle. Maimon's "Ideas of the Understanding."*

Maimon criticizes Kant in two very different ways. The first critique merely improves on Kant, whereas the latter takes a rigorous stance and insists on the antinomies of the circle's construction. These antinomies arise as soon as "motion" is scrutinized. It then turns out that "motion" obliterates the clear distinction between the understanding and intuition. But if motion is not allowed in construction, then a continuous line - and therefore also a circle - cannot be constructed. Maimon captures this and similar problems with the term "ideas of the understanding" (in allusion to Kant's "ideas of reason"). These ideas are concepts of objects which cannot be presented because they involve infinity and yet can be infinitely approximated. Kant criticized Maimon's ideas of the understanding and maintained that in geometry we do not need actual infinity (all the points of a line) but merely "any" assigned point. This objection is answered by Maimon's investigation into the question ofwhich theorems apply to "any" point and which to "all" points. The results of this investigation are important not only for geometry. They also show what property is a "proprium", i.e. one which is peculiar to a specific object and which conforms to Maimon's "Law of determinability." Thus e.g. if a theorem is valid for "any" assigned point on the circumference, then it is not a "*proprium*" of the circle because these points could also be the apexes of a polygon. A "*proprium*" of the circle can be only a property which belongs to the circle and to the circle only. The difference between the polygon and the circle reproduces the duality of definition and construction, of understanding and intuition. Maimon also attempts to show that this duality is inherent to human cognition and shows in inevitable antinomies. Finally, Maimon suggests alternatives to the construction by motion. He suggests constructing the circle (and other curves) by conic sections. This will be discussed later (#6).

3.4. *Constructing the Circle by Motion*

Consider first the more lenient critique of Kant's construction of the circle. The concept of the circle is given to the understanding in the form of the Euclidean definition (Tr, 50). This is its *essentia nominalis*. But we do not yet know whether the circle is possible in space (Tr, 39) To construct this object in intuition we apply the construction rule that prescribes turning a segment around one of its ends (Tr, 51). Finally, we prove that the constructed figure satisfies the definition.

The proof is analytic. Since the radius used to draw the circle is the same in all its possible positions, it follows "that it is identical with the concept of the circle (its conditions) (Tr, 43). This is so because the definition demanded that the distances between the circumference and the center be equal, i.e. the radii be equal - and this equality is guaranteed since the segment that produced this periphery "is equal to itself in all possible positions". (Tr 42-43; 50-51)

Now, Maimon's understanding differs in two points from Kant's. Maimon does not believe that the rule of construction follows from the definition nor, therefore, that the proof is superfluous.[87]

However, this method involves two major drawbacks: First, we have to introduce motion into geometry. The traditional objection is that this is not consistent with the rigor demanded of geometry. Motion refers either to the imagination or to mechanics. We can either imagine a geometrical object constructed by the motion of a point, or explicitly introduce further assumptions to justify our conclusions, i.e. that the radius remains invariant under motion and that the circumference delineated is continuous.[88] Since the construction rule is neither identical to the definition nor inferred from it, we must either first introduce these assumptions or prove that the object constructed "corresponds" to its definition. However, the proof can refer to a finite number of points only and these may be the vertices of a polygon,

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87. In a footnote Maimon remarks that the rule according to which a circle is empirically constructed is a "practical corollary" to the definition of a circle; he does not specify whether this is Euclid's definition or the "real definition" used e.g. by Kästner. Tr, 42; vgl. AA XX 53.

88. Maimon had good reasons not to presuppose that motion is continuous. See below # 4.1.)

not necessarily on the circumference of a circle.

Furthermore, is motion itself a priori o r aposteriori? And if it is empirical and yet indispensable for the construction of geometrical objects, how can geometry be a priori?[89] Finally, Maimon also says that the concept of the circle "had luck(!) that Euclid really invented a method to bring it into intuition a priori." (Tr, 50-51)[90] "Luck" means that the rule is not inferred from the definition and that there is not even a clear procedure to find a rule of construction when the definition is given. The rule of construction of the circle is a singular case and not a paradigmatic example for the construction of geometrical objects in general.

3.5. *Rigorous Construction: Circle and Polygon*

Maimon's second and more principal critique does not accept motion as a method of construction. The critique proceeds from Euclid's definition.

> “A circle is a plane figure contained by one line such that all the straight lines falling upon it from one point among those lying within the figure are equal to one another.” (*Elements* I, definition 15, Heath I, 153)

Suppose we use this definition of the circle as a rule of construction. The procedure is as follows: To take a “distance” (the Greeks had no word for “radius”) and to mark a number of equidistant points from the center. These points will all be on the circumference of the cir-

89. Maimon seems to believe that for Kant motion is a posteriori (Tr, 50-51, note; and explicitly: GW VII, 189). Indeed, in the first edition of *CpR* Kant maintained that motion involves experience (A 41). C.G. Schütz raised the objection that if so and under Kant's condition that geometrical objects must first be drawn by motion, then geometry cannot be a priori. See Schulz's review of Kants *Erläuterungen* and *Prolegomena*, Allgemeine Literatur-Zeitung, 162 (1785), p. 43, see Webb 1987, 38-39. In the second edition of *CpR*, Kant distinguished between the empirical motion of a material body and the a priori motion of a point or a plane describing space (B 155, note) (Cf. *Metaphysical Foundations of Natural Science*, *Theoretical Philosophy*, p. 202; Cf. also Friedman, 41-43) Maimon himself changes his mind on this point. In a footnote to chapter 2 he argues that motion must be as a priori as space is, since the intuition of space arises with motion (Tr 50-51, note). However, it is not absolutely a priori as are pure concepts: it is valid for objects in our forms of intuition only (Tr, 56). Later, in his "Antwort" of 1790 ("Antwort", 73) and again in 1793, Maimon counts motion under the empirical concepts and, moreover, all information added to the concept by intuition as dubious. "Intuition as such has no intellectual reality." (See "Über die Progressen der Philosophie", GW IV, 57)

90. The attribution of this method to Euclid is, of course, wrong, but was nevertheless common.

cle, but they do not yet form the “line” required. They have to be connected to form a line. It is, however the straight line which is uniquely determined between any two points. (Postulates 1 and 2 in *Elements* book I which guarantee the possibility of drawing a straight line and also imply that it is unique.) If all points equidistant from the center are thus connected, and if, to choose the simplest rule of construction, they are also equidistant from each other, we obtain a rectilinear figure, e.g. a regular polygon but not a circle. However, at all the points assigned (which are the vertices of the polygon) it indeed satisfies the definition of a circle. Hence the regular polygon turns out to be the instantiation of the concept (i.e. the definition) of the circle. Later Maimon will say that the regular polygon may be called the “concept of the circle”[91].

We can rigorously construct points that are equidistant from the center, but we can do this only for a finite number of points, and these may be the vertices of a polygon, not necessarily points on the circumference of a circle. The reason is the very same that frustrates the proof than an object constructed by the revolution of a segment around one of its ends is a circle. Neither a construction following Euclid's definition nor a proof can satisfy the requirement that *all* points of the circumference are equidistant from the center. The gist of the argument is that a polygon with any number of vertices is conceptually distinct from a circle although it may infinitely approximate it. The difference lies in that a (straight) line connecting its vertices is not equivalent to an arc of a circle (a curve). This discussion must have shown Maimon that his early proof that the straight line is also the shortest between two points is false, since it rested on the equivalence between the broken straight and the curved lines which he now rejected.

In the second chapter of Tr, Maimon discusses the gap between the concept (of the understanding) and the object (in intuition) using the example of √2 (Tr, 58-9). We have a precise concept of this number ("A number which multiplied with itself yields 2") (Tr, 58), and we also have the rule of its construction, and yet the entire number cannot be presented as a "definite number." Maimon therefore distinguishes between the "formal" and the "real" possibility of the object. √2 is possibile "formaliter" but not "materialiter". Maimon dubbed con-

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91. “Ein reguläres Poligon ist in Beziehung auf den Zirkel (in dem oder um den es beschrieben wird) Begriff”. (GW III, 186)

cepts of this kind "ideas of the understanding".[92] A further consideration shows that this is also the problem with the construction of the circle, which is discussed in the third chapter of *Transcendentalphilosophie.* The concept is given in both cases, but the object cannot be constructed because its construction involves infinity. Definition and rule of construction, *essentia formalis et realis*, understanding and intuition, remain distinct.

Maimon suggests, therefore, that we distinguish between "the totality of conditions by which an object of intuition is thought and the totality of the intuitions which are subsumed under these conditions." (Tr, 76, 75, 42, note). The first, the formal completeness, is a unity, a concept of the understanding, whereas real completeness is a manifold, an infinite number of intuitions that cannot be realized. And yet, concept and object are not simply disjunct since we can progressively reduce the difference between the concept and its object. Thus the real circle, the circle in intuition that satisfies the concept of the understanding, is a "concept of limit":

> "Hence it is not a concept of the understanding to which an object corresponds, but merely an idea of the understanding, which we can ever more approximate *ad infinitum* in the intuition by means of adding such lines, and therefore it is a concept of a limit." (Gränzbegriff) (Tr,75-76)

3.6. *Kant's Critique of Maimon's "ideas of the understanding"*

In his letter to Herz and Maimon, Kant criticized Maimon's notion of an "idea of the understanding" exemplified by the concept of a circle. Kant objected that "ideas of the understanding" may be dispensed with. The wording of the definition of a circle, namely that all distances between the points on the periphery and the center are equal, actually means that "any"

92. Thus there are series which converge to a limit in infinity and which can therefore be substituted by their limit value, and vice versa, this value may be replaced by the corresponding series. Now, it is important that the same mathematical entity is here given both as an object (a value) and as a rule of construction (of the series). Moreover, when we understand the law of the series, we know both the value to which it converges (and is as such given as an object) but also that this value cannot be reached by construction (and therefore never be given as an object).

Here reason runs into an antinomy in that it prescribes a rule according to which we certainly must find [the idea] and at the same time demonstrates that it is impossible to realize it. (Tr, 229) chosen distance is equal to "any" other, not that "all" infinitely many distances are equal.[93] Otherwise, Kant continues, we could also maintain that every line is an idea of the understanding, for also on a line there are infinitely many segments between any two points. Concerning the circle, we need not draw all radii and infinity is not involved.

However, in the *Critique of Pure Reason*, when discussing the concept of "definition", Kant wished to dispense with the predicate "curved" in the definition of a circle ("A curved line every point on which is equidistant from ... the centre") , his argument being that "curved" follows "if all [sic!] points (alle Punkte) in a line are equidistant from one and the same point". (B 759-760, see above # 3.2). When alerted here to the problem, Kant maintained that only "any" point was required

But if we do not distinguish between "any" of the points actually assigned and "all" points, then a polygon is not conceptually distinguished from a circle. Both satisfy the following definition: "A plane figure contained by a (?!) line such that the line falling from any point assigned on the periphery on one point among those lying within the figure is equal to any other."

Kant does not distinguish hence conceptually between a "straight" and a "curved" line, between a polygon and a circle. The difference can be diminished with the number of points assigned until both lines coincide. In the case of the polygon the number of points equidistant from the center is finite, in the case of the circle infinite. This approach was rather common as Christian Wolff's definition of "curve" shows:

> A curved line is that line the parts of which are not similar to the whole line, or which can be distinguished from it. In the new geometry we imagine the curved lines as if compounded of infinitely small straight lines. A straight line is described by a point that continuously keeps the same direction (Richtung), and therefore all points on a straight line lie in the same direction (Gegend). A curved line is however described by a point that changes its direction (Richtung) continuously (stets). But since it must keep the same direction for a short while - since otherwise

93. "eine jede Linie, nicht das All der Linien." (AA XI, 53).

> the latter could not have been changed - a straight line is described in this very same short time. Hence in the new geometry we suppose that a curved line is a many-sided polygon of infinitely many and infinitely small sides.[94]

Traditionally, lines were divided into classes, whether into "straight" and "curved", or "in-composite" and "composite" or other possible divisions and subdivisions.[95] On this basis, the predication of a line that it is both "straight" and "curved", or "composite" and "in-composite" certainly constitutes an inconsistent statement. To predicate of a line that it is both straight and curved, that it is "a straight-curved line" as it were, is a contradiction, says Maimon (GW VII, 142). Now, Kant evidently accepts the modern concept of a curve and sees no reason to engage in the classification which produces the dichotomy between "straight" and "curved". Maimon obviously does not accept the composition of a curve out of infinitesimal straight lines as a satisfactory solution of the conceptual problem. Does the difference imply implications that are relevant to geometry?

3.7. *Maimon's Rebuttal of Kant's critique*

Two questions arise. Do we need in geometry "all" points of an object or only "any" of the points actually assigned? Do we need the conceptual distinction between a "curved" and a "straight" line? In answer to the first question, Maimon attempts to determine on what occasions we need "all" points on a line and for what purposes "any" or even a small number of such points would do. As to the conceptual problems, Maimon's position is that indeed the

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94. "Eine krumme Linie wird genennet, deren Tehile der ganzen Linie nicht ähnlich sind, oder sich von ihr gar wohl unterscheidenlassen. Man stellet sich in der neueren Geometrie vor, als wären die krummen Linie aus uunendlich kleinen geraden Linien zusammen gesetzt. Denn eine gerade Linie wird beschrieben von einem Puncte, der beständig einerley Richtung behält, daher alle Puncte in einer geraden Linie gegen eine Gegend liegen. Hingegen eine krumme Linie wird von einem Puncte beschrieben, der seine Richtung stets ändert. Weil doch aber eine kleine Weile seyn muß, da er einerley Richtung behält, denn sonst könte sie nicht geändert werden; so wird in selbiger kleinen zeit eine gerade Linie beschrieben. Daher setzt man nun in der neueren Geometrie, es sey die krumme Linie ein Viel-Ecke von unendlich vielen und unendlich kleinen Seiten." (Wolff, Mathematisches LexiconWolff, Christian: Mathematisches Lexicon. Hrsg. und bearbeitet von J.E. Hofmann. Hildesheim und New York 1978, Sp. 749, "curva", Sp. 460-461). Very similar also Käster, pp. 160-162.

95. See above # 2.8.

concept of the infinitesimal implies unresolved antinomies. However, he also believes that antinomies are inherent to our finite understanding. He therefore does not attempt to mitigate them, nor to exclude them by banning the concept of the infinitesimal. Maimon rather places this concept in the center of his thought and uncovers and emphasizes the arising antinomies as essential to human thought.

Do we need in geometry "all" points of a figure or merely "any" point? Maimon maintains that some theorems of geometry indeed refer to "any" point assigned on the circumference, others to "all" thus that the latter concept is indispensable.

Consider first the straight line. The definition demanded that "all" its segments lie in the same direction. Kant maintained that this should be understood to say that "any" segment must conform to this condition, Maimon that "all" are required. In the printed version of Tr, Maimon addressed Kant's objection - again without naming him[96] - and repeated his distinction between the completeness of the construction rule and the completed construction:

> "Notwithstanding their material incompleteness, these concepts or rather ideas of the understanding are nevertheless accurate because we can understand their rules by whatever is actually given in intuition. Their material completeness requires continuous (beständig) repetition of [the application of] these very rules. But since, according to the conditions of the rules, this repetition must be infinite, they remain mere ideas and have in their application the same grade of accuracy (Richtigkeit) as the grade of their material completeness. For example, the axiom: "A straight line is the shortest between two points" is, when applied to a given line, so much more accurate the more straight parts we observe in it." (Tr, 80)

We can now appreciate the import of Maimon's distinction between the "formal" and "material" completeness of mathematical definitions. The construction of a geometrical object necessarily involves continuity because geometry is the science of continuous magnitude. Continuity involves, on the one hand, either an infinity which cannot be completed or , on the other

96. Tr, 79-80; cf. Kant XI, 53

hand, the introduction of motion or intuition which cannot be understood. Although a geometrical object can be practically approximated to any degree desired, its concept and its construction remain conceptually distinct as ever. The gap between the concept and the object can be bridged either by motion (and this, again, is doubtful as the discussion of the *rota Aristotelis* below will show) or by approximation. Both methods belong to intuition which is opaque to reason. Thus the difference between a polygon with innumerable sides and a circle may not be recognized in intuition, and also the difference in their areas may be rendered smaller than any given magnitude (*omni dabili minor*), but conceptually they remain disjunct. But if this is so, then the alleged "apodictic" truth of geometry depends on intuition and is, therefore, merely of subjective, not objective necessity. We are compelled to give our assent to a proposition although we do not understand why this is so. Geometry is imposed on us, as are also empirical perceptions.

However, we do not always need *all* points of a circle. Consider the proposition that every perpendicular drawn from the periphery of the circle onto its diameter is the middle proportional between the two segments of the diameter it assigns. Let the perpendicular be c and let the respective segments of the diameter be a and b, it follows that c:a :: b:c.[97]

Maimon commentary at this point is:

> “Here we do not need to suppose that all lines which can be drawn from the center [to the periphery] are equal, but only three of them.” (Tr, 78; see Tr, 79).

These three points determine in their turn a circle.[98]

However, it is not generally the case that we need only a limited number of points on the circumference of the circle. For propositions pertaining to what Maimon will later call the "measurability" (Ausmeßbarkeit) of an object, not single points, but the entire continuous ob-

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97. This follows from *Elements* VI, 13 and III,1.

I guess that Maimon's example alludes to Kant's Prolegomena, # 38. Kant there refers to *Elements* III, 35. This proposition, too, refers to the ratio of segments cut from any two intersecting chords in a circle. In that proposition merely four points on the circle's circumference are needed. Maimon's example is a special case of III.35.

98. This follows from an inversion of *Elements* III,1 with the relevant porism

ject is required. Since geometry is the science of continuous magnitude, a geometrical line must satisfy two requirements: it must be a continuous magnitude, and it must be a measurable, i.e. a geometrical magnitude ("Antwort", GW III 195-196). Only the straight line immediately satisfies both requirements, all others satisfy only one. The definition of a curve (expressed in an equation), determines any number of *loci geometrici* of this curve, but not the continuous line itself ("Antwort", 68), and only these *loci geometrici* satisfy the ratios defined by the equation ("Antwort", 71-72). This line is not measurable as a continuous magnitude. Maimon sides therefore with the ancient mathematicians against the moderns, especially Descartes.[99] He quotes the latter's "astonishment" that the ancients excluded from geometry those curves which cannot be constructed with a ruler and a compass (so-called "mechanical" curves) but he does not share this astonishment - on the contrary.[100]

In the case of the circle, we have propositions which refer to the entire circumference or the entire area, and are true of the completed circle only. Consider e.g. the theorem concerning the ratio of the arc to the diameter of a circle.[101] Here we do not refer to single points on the periphery of the circle but to the entire area enclosed within the circle. Here “we must consider the circle as already completed, since otherwise this ratio [of its area to the area of a square] will not be exact.” (Tr, 78)[102]

In order to construct a real circle (as distinguished from a polygon), we must first turn to intuition and use motion (by the revolution of a segment around one of its ends), then prove that the object constructed is a circle and suppose that "motion" is a priori.[103] However,

99. Maimon quotes Descartes' Geometry II.2, “Antwort des Herrn Maimons auf voriges Schreiben” Berlinisches Journal für Aufklärung (October 1790), pp. 52-80, reprinted in the article "Wahrheit" in: Philosophischhes Wörterbuch, GW III, 185-202; I quote according to this text, the quotation above: GW III, 195. The text was reprinted again in *Transcendentalphilosophie* (2004), 239-251.

100. Note that the compass serves to apply a distance, not to construct a circle.

101. According to Euclid's *Elements* XII, 1-2.

102. As we will see later, maimon also maintained that in order to know a priori that the area of the circle can be measured it must be considered as a polygon. The assertion that the area of the circle can be exactly calculated implies therefore that "circle" is understood both as a polygon and as not a polygon.

103. “Beschreibt man hingegen einen Zirkel durch Bewegung einer Linie um einen ihrer Endpunkten, alsdann wird die Konstrukzion dem Begriffe völlig entsprechen.“ (GW III, 194)

this does not suffice to know a priori all properties of the circle. Consider e.g. the measurability of the area of the circle. How do we know that this area can be measured?

> In the same way we know the possibility of measuring the circle i.e. comparing the content of its enclosed plane area with an area enclosed in straight lines and which serves as a unity, and also the necessity of the following proposition: The enclosed area of a given circle equals that of a triangle whose height equals half the diameter and whose basis equals the circumference of the circle. We learn these propositions by proving according to the method of exhaustion that the concept of the circle contains the general concept of the polygon and must be measurable as this one is. The measurability of the circle is, therefore, known a priori before the determination of what is peculiar to it is known (i.e. that it is a polygon of infinitely many sides) by the measurability of what is general, which is likewise contained in it [in a polygon in general].[104]

Maimon thus not only substantiated the fact that if used as a rule of construction the definition of a circle produces a regular polygon, but also maintained that in the limit, when the polygon coincides with the circle, it still retains the properties of a polygon. The area of a circle is not only calculated as if it were a polygon but we know a priori that it is measurable because it is a specific kind of a polygon. And yet, the calculation of the area will be exact only

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104. "Ueber den Gebrauch der Philosophie zur Erweiterung der Erkenntniß", 1795, VI, 362-396, here: 368. This argument was not an original idea of Maimon's. Consider for example the following remark of Christian Wolff: "Das Prinzip der Reduktion nenne ich das Kunstmittel, durch das man ein Objekt, bei dem etwas gefragt ist, auf ein anderes zurückführe, welches einen gemeinsamen Begriff hat, so daß das, was von diesem uns bekannt ist, kraft des allgemeinen Begriffs auch auf jenes angewendet werden kann; - der Kreis selbst ein Polygon, um seine Fläche zu finden." Chr. Wolff, Psych. Emp. § 472.

The problem was raised by Locke: "We have the ideas of a Square, a Circle, and Equality; and yet, perhaps, shall never be able to find a circle equal to a square, and certainly know that it is so" (Essay IV.iii.# 6, p. 540).

Leibniz answred with reference to Archimedes (*Circuli dimensio*). Maimon's answer is much more radical. He does not merely maintain that a circle can be approximated by a polygon, but he grounds this possibility in the essential definition of the circle: a circle is (and is also not) a polygon.

if the circle is not a polygon. The Archimidean method of exhaustion is here interpreted not a rule of the thumb and a makeshift for practical purposes but as conceptually justified because a circle really is (and is not) a polygon.[105]

This duality is reproduced in the corresponding rules of construction. We know (a priori) that the area of a circle may be calculated because we consider the circle as a polygon (with infinitely many sides). Turning the definition of a circle into a rule of construction, we assign any number of equidistant points from the center and connect them. We obtain a polygon. Each side of the polygon thus constructed can be considered as the basis of an isosceles triangle of which the circle's center is the vertex. Since we know that and how we can calculate the area of a triangle, we also know that we can calculate the area of a polygon. However, in the case of the circle-polygon the more sides this polygon has, the more "exact" will our calculation be i.e. the better will it approximate the area of the circle. It will be "exact" if we construct a polygon with infinitely many sides, i.e. a circle.[106] However, we can construct a circle only if we do not use its definition as a rule of construction, but by turning a segment around one of its ends. Of course, the requirements to construct a circle (by means of the revolution of a straight segment around one of its ends) and considering it as if it were a polygon (constructed according to Euclid's definition of a circle by assigning a finite number of points which are equidistant from the center), exclude each other - and yet are both indispensable. This is the topic of Maimon's later considerations.

It is easy to "see" that a polygon with infinitely many sides coincides with a circle, but conceptually they are contradictory. The dichotomy reproduces the problem encountered in the proof that the straight line is the shortest between two points: any two points on the polygon are connected by a straight line; any two points on the circle are connected by an arc. And yet, they should be treated as both equivalent and not-equivalent when the circumfer-

105. It seems that in analytic geometry the construction of a circle has nothing to do with polygons. The equation $(x-p)^2 + (y-q)^2 = r^2$ does not use existing geometrical figures to construct the circle. Maimon, however, maintains that the equation merely determines for whatever values of x and y the value of r. This value determines in its turn any number of points on the circumference of the circle to be constructed. Hence the algebraic equation determines only the radius which in the synthetic version was given. From here on the construction is identical in both forms - and so are the arising problems. (See GW III, 194)

106. GW VI, 368

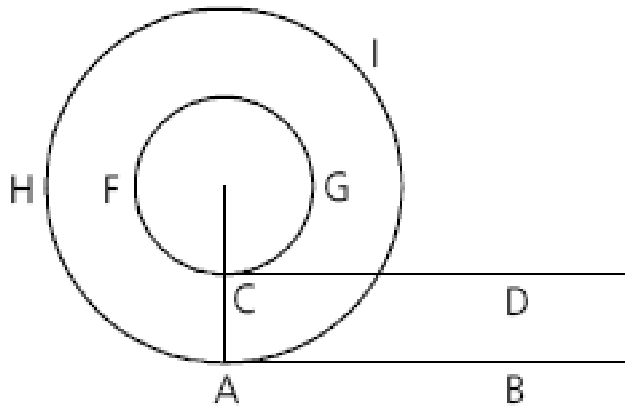
ence and the area are measured. In all propositions concerning the measurability of geometrical objects Kant's suggestion to consider "any" point assigned and not "all" points fails. Maimon rather integrates the antinomies that arise into his philosophy.

3.8. *Rota Aristotelis. Antinomies*

Maimon's discussion of the *rota Aristotelis* is presented as an argument for his conception of antinomies and against Kant's. Kant maintained that the antinomies of reason arise when reason attempts to think the absolute as an object. Kant's critique shows that the unconditioned, the absolute or the infinite cannot be given in experience, i.e. in the realm of appearance. Antinomies are resolved by the critique and the ensuing distinction between appearance and experience on the one hand and the "thing-in-itself" and dogmatic speculation on the other. For whatever is given in experience the next condition or consequence can be sought, and therefore this process can and must continue in infinity. The antinomies hence arise because actual infinity is supposed to exist in the world of experience where only potential infinity is meaningful. When this is understood, the source of the antinomies is found and the restriction of knowledge (Erkenntnis) to experience does not allow the antinomies to rise.

In the *Transcendentalphilosophie* Maimon discusses the antinomies in a chapter entitled "Antinomies. Ideas". He first summarized Kant's position and then opposed to it his own view. In contradistinction to Kant's view, Maimon claims that reason "can and must" be considered both as limited by sensibility and yet also as unlimited and absolute. Maimon hence believes that reason cannot be constrained to the realm of sensibility to which Kant wished to limit it, but that "absolute" reason is an aspect of human experience that cannot be disciplined. The dichotomy between appearance and thing-in-itself is not accepted by Maimon and whereas Kant looked for a demarcation in order to exclude antinomies, Maimon accepted them as inevitable in human thought.

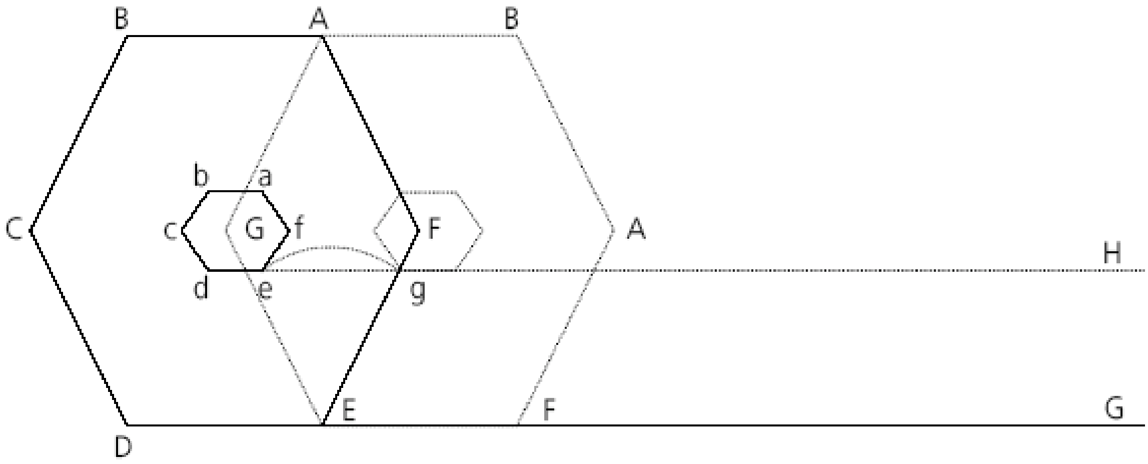
The problem of the *rota Aristotelis* consists in this: Two concentric wheels are placed one within the other and rigidly connected. Let their centers be connected by a vertical line and draw the tangents at the points of intersection of this vertical with the circles. After a complete revolution, the larger wheel will have covered on its tangent a segment equal to its circumference such that all parts of the circumference consecutively touch all parts of the line. The same must be true of the inner, smaller circle on its own tangent. However, the smaller circle will have covered an equal distance to the larger one although its circumference is smaller.



Maimon first presents the antinomy of the *rota Aristotelis* (Tr 231-232), then he quotes in extenso Kästner: *Anfangsgründe der Analysis endlicher Grössen*, § 601[107] where the problem is resolved following Galileo's method[108]. Maimon then ventures to interpret Kästner, allegedly because Kästner did not supply a figure. Maimon supplies a figure (Tr, 234) which follows Galileo's (Drake 29), but is not identical with it. His discussion essentially follows Galileo's. After criticizing Galileo's and Kästner's resolution of the antinomy, Maimon offers his own version of it and why it should be acknowledged and accepted as such.

107. Siehe Abraham Gotthelf Kästner, Anfangsgründe der Analysis endlicher Grössen, Göttingen (Vandenhoek) 1760 (21767), § 601

108. See Galileo, Two New Sciences (1638), transl. and introduced by Stillman Drake, pp. 29ff, 56-57) (Tr. 232-234)



In Galilei the circles are substituted by regular polygons, here by hexagons. Now, whereas the sides of the larger hexagon continuously cover the straight line, the sides of the inner, smaller hexagon do not. Each of its side first “jumps” without touching the tangent before it again touches it and covers a part of it. Clearly, in a rigid wheel, these "jumps" are physically impossible.[109] Nevertheless, the sides of the large polygon cover their distance continuously, whereas those of the small one do not.[110] The distance discontinuously covered by the sides of the small polygon consist of the lengths of the sides of the polygons plus the distances “jumped over”. These latter distances are the chords of the arcs described by the polygon’s vertices before they again touch the line. This explanation should hold also for a polygon with infinitely many and infinitely small sides, i.e. for a circle.

Maimon contribution consists in the following two points. First, he criticizes the substitution of polygons for the circles. Second, he criticizes Galileo’s (and Kästner’s) solution and suggests that the antinomy be accepted as real. In his view it is but another instance of the inevitable “antinomies of thought”.

The antinomy arises because the two polygons have different perimeters and because in an independent complete revolution each of them would cover a distance equal to its own perimeter. And yet, when the smaller is placed within the greater and rigidly connected with

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109. Maimon discusses this question in his commentary on Mamonides' *Guide*. See my discussion below.

110. “Die Theile des äussern Polygons ABC usw. decken nach und nach die Linie DG stetig; hingegen die Theile des innern Polygons abc usw. decken die Linie dH nicht stetig” (Tr 234-5)

it, the distance between the verticals connecting their centers at the beginning and end of a complete revolution will be the same. It thus seems that their perimeters must be of equal and unequal length at the same time.

Maimon objects that there is no reason to measure the distance covered starting with the common center. The choice of this point was justified in the case of the circles because at each instant of time they touched their tangents at one point only and the distance covered was measured between these points, namely the intersection of the tangent and the vertical going through the centers (Tr, 236). However, the polygons cover the tangents each time with the full length of their sides and the choice to measure the distance covered between their centers at the beginning and end of a complete revolution is not justified in this case. Rather, the distance should be measured between the opposite ends of the first and last sides covered by the rotating polygons. Since the distances between these ends are exactly equal to the perimeters of the polygons, no problem arises and there is no antinomy to resolve.[111]

Now, in the case of real circles (not polygons), the antinomy arises but, according to Maimon, cannot be resolved. It comes about because the smaller and larger circles touch the same number of points and yet cover different distances. Since the circles are concentric we can draw through each point on one of them a radius assigning exactly one point also on the other. Therefore, the smaller and larger circles touch the same number of points on their tangents. And yet the circles cover distances of different length. Kästner said that this is not a contradiction "since lines are not the sums of points" (Tr, 233).

This resolution of the antinomy is again criticized by Maimon. On the one hand Kästner considers the sides of the polygons as infinitesimally small such that the polygons may be considered as circles (and therefore the distance covered is measured between the singular points of contact of each circle with its tangent). On the other hand the sides of the polygons must have determined magnitudes such that the sum of the sides "jumped over" in the revolu-

111. Maimon restricts here (and later) this method of excluding the antinomies to polygons. However, in the next sentence he speaks of circles too: The paradox does not arise "as long as it cannot be proven that the circumference of the smaller circle plus the difference between the circumferences of the greater and smaller must be smaller than the circumference of the larger one." (Tr, 235) Since he later says (236) that neither Kästner's nor his solution work when a polygon of infinitely many sides is considered, I believe that "circles" here is a lapsus calami.

tion may be equal to the difference between the perimeters of the large and small polygon.

> "If the sides are infinitesimally small, so must be the aforementioned arcs and hence also their chords; nevertheless these chords taken infinitely many times should be equal to a finite line (namely the difference between the perimeters of the larger and smaller circle)." (Tr, 236)

Maimon hence constructed a typical dilemma to refute Galileo and Kästner. If we consider polygons which are different from circles, then there is no justification to measure the distance covered between the centers at the beginning and end of a complete revolution and the antinomy does not arise in the first place. If we consider polygons which are not different from circles, then the antinomy arises, but its resolution by means of the segments "jumped over" is invalid and the antinomy remains unresolved.

3.9. *Maimon's View of the Antinomy*

Maimon himself opposes a "real" to a "merely mathematical" infinite and accordingly a "physical" and "mathematical" antinomy. A "real" infinite is an actually existing infinite plurality such as the number of particles of an existing physical body; a "mathematical" infinite is a potentially infinite plurality to be constructed. Because these concepts are incompatible and reason nevertheless "commands" both, a "true" antinomy arises:

> We must hence admit a real infinite (not merely a mathematical infinity, i.e. the possibility of dividing in infinitum) as the element of the finite. Hence a true antinomy arises here because reason commands us (by the idea of the divisibility of space in infinitum) never to stop the division of a certain line, such that we finally do [not] arrive at an infinitely small part, and yet at the same time it demonstrates, that in this case we must really run into such an infinitely small part. (236-237)[112]

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112. "Wir müssen also ein wirkliches (nicht bloß mathematisches, d.h. die Möglichkeit der Theilung ins Unendliche) Unendliches, als das Element des Endlichen zugeben. Es ent/springt also hier eine whare Antinomie, indem die Vernunft uns (durch die Idee der Theilbarkeit des Raums ins Unendliche) befiehlt, mit der Theilung einer bestimmten Linie niemals aufzuhören, so daß wir zuletzt auf [k]einen unendlich kleinen Theil gerathen, und doch demonstrieret sie uns zugleich, daß wir im vorgelegten Falle auf einen solchen kleinen Theil wirklich gerathen müssen." (236-37)

This antinomy corresponds of course to Kant's "second antinomy" in the *Critique of Pure Reason*, concerning the "Absolute completeness in the Division of a given whole in the [field of] appearance." (*CpR* 443) However, Maimon's understanding and interpretation of it is entirely different from Kant's. Kant discussed the "division of the given whole" that cannot reach a limit, whereas the extension of material bodies demands that there be simple parts from which they are composed. What Kant conceived as a "mathematical" antinomy is conceived by Maimon as a "physical", not merely mathematical antinomy. The "mathematical" antinomy can be resolved by Kant's "system of sensibility and its forms", the "physical" antinomies cannot. Kant argued that since space is a form of our intuition and not an object in itself, it is given as a whole and it is not in itself composed of parts, although it can be divided into parts.

> Space should properly be called not *compositum* but *totum*, since its parts are possible only in the whole, not the whole through the parts. It might, indeed, be called a *compositum ideale*, but not *reale*. (*CpR* B 466)

It therefore makes no sense to speak of its composition prior to our division of it. The antinomy arises because we consider space as if it were an object in itself and composed of infinitely many and infinitely small parts, and yet maintain that we can divide it in infinitum - with the antinomy following from both these notions. When we remember that space is a subjective form of sensibility given to us as a *totum* and not compounded of parts, the antinomy does not arise. And conversely, if material bodies were given to us independently of the subjective forms of intuition, if we experienced the "things in themselves" and not only "appearances", then "The argument of the monadists would indeed be valid." (*CpR* B 470).

This is exactly Maimon's position. He does not accept the disjunction between "things in themselves" and "appearances", and therefore he accepts Kant's solution as valid for space, but not for material bodies. Space is a subjective form of sensibility, given as a totum, and it makes little sense to ask how it is composed independently of our division. The case is different with objectively real objects and processes as they appear in physical experience. Here it

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I corrected the obvious mistake "[k]einen."

makes little sense to say that if reason transcends its limits it will run into "dialectic" and "antinomies". Since we experience real wheels turning on straight surfaces, the problem of the *rota Aristotelis* is real. The antinomy does not arise because (as Kant said) "philosophy here plays tricks with mathematics" because it "forgets that in this discussion we are concerned only with appearances and their condition." (*CpR* B 467) Both inconsistent views of the wheel as a circle and as a polygon are necessary to account in mathematical terms for the physical phenomenon. They do not arise because a metaphysical principle is turned against mathematics. This is why Maimon names Kant's "mathematical" antinomy "physical". Maimon replaces Kant's disjunct “realms”, “appearance” and the “thing in itself”, with a continuum of knowledge on different levels because we can approach the circle ever more by adding smaller sides to the polygon. This antinomy of the *rota Aristotelis* points to a real duality which cannot be overcome, not merely to a mistake in determining the reference of our concepts. The antinomy is between “absolute” understanding (in which we nevertheless partake) and our understanding, limited as it is by sensibility. (Tr, 227) It is easy to see that this is the position of Leibniz, not of Kant.

The duality of circle and polygon shows up in more than one way in the antinomy before us. First, for us circles are given or constructed in intuition, we cannot construct the circle according to its definition. We can also think the polygon with any number of sides and understand that it infinitely approaches the circle. But we cannot conceive the coincidence of both. We therefore have to think the circle and the polygon with infinitely many and infinitely small sides as identical and not identical at the same time.

There is a general lesson to be drawn from these examples. The duality of construction and object, of potential and actual infinity cannot be overcome because we need both. Thus e.g. there are series which converge in infinity and which may therefore be substituted by their limit value. The same mathematical entity is presented to the understanding both as an object (a value) and as a rule of construction (a series). Moreover, when we understand the law of the series, we know both the value to which it converges (and as such is given as an object) but also that this value cannot be reached by construction (and therefore never be given as an object to intuition).

> Here reason runs into an antinomy in that it prescribes a rule according
> to which we certainly must find [the idea] and at the same time demon-

strates that it is impossible to realize it. (Tr, 229)[113]

Maimon does not refer here to a special class of objects but to different ways of thinking objects according to the finite or absolute understanding involved. The classical example of potential infinity is the series of natural numbers. This series can be thought of both as potential and as actual infinity.

> To us, the complete series of all natural numbers is not given to intuition as an object, but merely as an idea, by which we conceive the successive progress in infintium as an object. (Tr, 226-227).

This is so, because we cannot produce this series other than successively in time. But to absolute understanding, which is not limited by sensibility (time and space), the complete series of natural numbers may very well be given as an object.

> But absolute understanding thinks the concept of an infinite number without temporal succession, at once (auf einmal). Therefore, what is conceived with regard to the limitation of the understanding as a mere idea, is with regard to its absolute existence a real object. (Tr, 228, 237)[114]

The consequence is that we cannot ascribe one aspect of the understanding to God and another to humans, but that we have to accept this as the dual character of our own understanding:

> Our understanding must be looked at in two opposed respects: 1) as an absolute [understanding] (not limited by sensibility and its laws). 2) As our understanding, in respect to its limitation. Therefore [The understanding] can and must think its objects according to two opposed laws. (Tr, 227)

Accordingly, Maimon does not wish to resolve the antinomies (as Kant does), but rather acknowledges and accepts them as part of the *condition humaine*. What can and should be done

113. Maimon also claims (Tr, 228) that we can substitute a convergent series for a value. This is problematic as they may be more than one series that have the same limit.

114. This dual consideration of the series of natural numbers was not understood by A. Fraenkel. See his interpretation in GM 151.

with incessant labor is to increase the share of the understanding in human knowledge as compared with intuition and sensual experience, to continuously approach the *animal rationale* and remove oneself from the *animal sensuale*. These two incompatible respects do not form a dichotomy as in Kant. As Maimon indicates with the example of irrational roots and other recurrent series, finite reason may infinitely approach infinite reason. One can hardly conceive a greater distance to Kant on the same gound of the oppostion between “idea” and “object”.

Finally, the distance to critics of the calculus in the seventeenth century should also be noted. Maimon does not criticize the calculus. On the contrary. The differential calculus is in his view a "sparkle of divinity" and testifies to the descent of Man from the "pure intelligences". However, since we are finite we have to accept that partaking in the infinite understanding entangles our finite minds in antinomies. (*Versuch einer neuen Logik*, 1794, GW V, 266f)

4. *The Distrust of Intuition*

4.1. *Asymptotes*

Maimon's accepted the antinomy that a circle has to be conceived both as a circle and as a polygon. This was the result of his study of the definition and construction of the circle, on the one hand, and of Galileo's resolution of the antinomies of the *rota Aristotelis* on the other hand. However, the problem of the *rota Aristotelis* was not discovered by Galileo. As the name *rota Aristotelis* suggests, the problem had been known since antiquity. Long before he became acquainted with modern philosophy, Maimon, too, knew the antinomies through the *Guide of the Perplexed* of Maimonides. In his commentary on the first part of this book, Maimon discussed these problems. In the *Guide* I, 73, Maimonides discusses the "premises" of the "dialecticians" (the Mutakalimun), namely that bodies are composed of indivisible particles, that vacuum exists and that "time is composed of instants". Now, since in motion the distance traversed, the time elapsed and the (uniform) velocity are proportionate to each other, it follows that "no movement can be more rapid than another movement," since in all motion one unit of distance is covered in one unit of time (Pines, 197). The apparent discrepancies between the velocities of bodies arise from a greater or smaller number of units of rest in-between those of motion. Motion is hence not continuous. It appears continuous to the senses, but in truth, as conceived by the understanding, it is not. The example adduced as an argument against the Mutakalimun is a millstone making a complete revolution. Here the antinomy of the *rota Aristotelis* arises:

> "Has not the part that is at its circumference traversed the distance represented by the bigger circle in the same time in which the part near the center has traversed the distance represented by the smaller circle?"

The Mutakalimun answered the challenge maintaining that

> "in turning, the parts of the millstone burst, and the number of [instants of] rest of the parts close to the center exceeds the [number of instants] of rest of the more distant parts."[115]

Maimon comments on this sentence thus:

> The commentator spoke: Although this answer seems at first repulsive to the commonsense, it is nevertheless true in itself, if this bursting is not assumed to actually occur but only to be [so] conceived by the intellect. (GM 129)

Preferring the understanding to the senses, Maimon here sides with the Mutakalimun against Maimonides, but he sides with Maimonides against the Mutakalimun in the determination of what is possible and what is impossible. Maimonides prefers conceivability to the testimony of the senses accepted by the Mutakalimun as the criterion of possibility.

Maimonides accuses the Mutakalimun that they "are of the opinion that everything that may be imagined is an admissible notion for the intellect", and impossible "everything that cannot be imagined."[116] Here too Maimonides, the rationalist, wishes to replace the testimony of the senses with the judgment of the intellect. His example is the leg of an hyperbola and its asymptote which is the outline of the cone itself. These lines

> between which there is a certain distance at the outset, may go forth in

115. *Guide* I, 73; Pines, 197. I translated the phrase from the Medieval Hebrew translation of Ibn Tibbon which is bolder than Pines' translation from the Arabic.

116. *Guide* I, 73, tenth premise; Pines, 206-207.

> such a way that the farther they go, this distance diminishes and they come nearer to one another, but without it ever being possible for them to meet even if they are drawn forth to infinity and even though they come nearer to one another the farther they go. This cannot be imagined and can in no way enter within the net of the imagination." - And yet it is true.[117]

In his commentary on this argument, Maimon produces a three pages discussion with a simplified version of Apollonius' proof accompanied by a diagram. He was evidently very proud of this proof.[118] Both Maimonides and Maimon emphasize in this context that it is not by the imagination than Man is distinguished from the beasts but by the intellect.[119] Maimon adds to his interpretation some "Kantian" clarifications:

> The "concepts of the intellect are true, [and refer to] actually existing objects outside the intellect (Noumenon), but the concepts of the sense are merely visibles (phenomenon) as I explained. And therefore they are also dependent on the limitations and flaws of the intellect."[120]

Maimon here very clearly not only sides with Maimonides the rationalist against the Mutakalimun, but also with Leibniz against Kant: Leibniz, too, uses the same geometrical example and repeats Maimonides' argument in almost the same words. Moreover, in another context Leibniz praises Maimonides precisely because he determined the concept of possibility by criteria of the understanding, not the imagination.[121] Just as Maimonides' concept of

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117. *Guide* I, 73, Pines 210. See Apollonius, Conic Sections, Book II, theorem xiii or II, 1,2,14. The "two lines" are the hyperbola and its asymptotes.

118. GM 146-148. In his autobiography, Maimon's remarks that he proved Apollonius' propostion independently of "curved lines". Indeed, his section is perpendicular to the base and parallel to the axial triangle of the conic. (GW I, 381)

119. *Guide* I, 73; Pines 209; GM 142-143.

120. GM 142. The words "noumenon", "phenomenon" are inserted in brackets in Hebrew letters.

121. See Leibniz, *Nouveaux Essais*, IV, 12, #4. It is noteworthy that it is exactly this rationalist concept of possibility that Leibniz praises in his notes on the *Guide*. "Maimonides clearly distinguishes throughout between reason and imagination and teaches that the former, not the latter must judge as to the matter of possibility." G.W. Leibniz, Observations on Rabbi Moses Maimonides' book entitled Doctor Perplexorum, in: A. Fourcher de Careil, *Leibniz, La philosophie juive et la Cabale*, Paris 1861, p. 45. English translation by Lenn E. Goodman, "Maimonides and Leibniz,

segula prepared Maimon for the discussion of "synthetic judgment a priori," Maimonides' discussion of the asymptotes was known to him long before he encountered it in Leibniz; and, in general, Maimonides' rationalism prepared him for Leibniz's view that perception is but confused thought. This outlook had a double critical result concerning Kant. First, Maimon saw no reason to accept the dichotomy of "appearance" and "thing in themselves", but rather replaced it with a continuum. Moreover, there is also no synthesis of the understanding and intuition. On principle, the intellect can know Truth of "things-in-themselves", and the infinite progress of knowledge consists in replacing perceptual knowledge with clear concepts of the understanding.

4.2. *A Three-Lateral Figure Has Three Angles: Hic volo, hic iubeo*

The distrust of sensibility clearly appears in Maimon's criticism of an example by which Kant wished to demonstrate how synthetic knowledge a priori is produced in construction. Kant opened the second edition of the *Critique of Pure Reason* with a laudatio on construction in mathemaitcs. The example is Euclid's first proposition, namely the construction of an equilateral (Kant erroneously writes: isosceles) triangle.

> A new light flashed upon the mind of the first man (be he Thales or some other) who demonstrated the properties of the isosceles triangle. The true method, so he found, was not to inspect what he discerned either in the figure, or in the bare concept of it, and from this, as it were, to read off its properties; but to bring out what was necessarily implied in the concepts that he had himself formed a priori, and had put into the figure in the construction by which he presented it to himself. If he is to know anything with a priori certainty he must not ascribe to the figure anything save what necessarily follows from what he has himself set into it in accordance with his concept. (*CpR* B XI-XII)

Now, what are the properties "necessarily implied in the concepts that he had himself formed a priori, and had put into the figure in the construction"? Kant discusses on one occasion the proof that the sum of the internal angles of the triangle equals two right angles. In a different

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Journal of Jewish Studies, XXXI, 2 (Autumn 1980), pp. 214-236, here: 236.

context, Kant named a property immediately following from construction and said not only that it was synthetic a priori but also that it demonstrated the sovereignty of reason:

> "Thus, for example, the proposition: Any three-sided figure has three angles (*figura trilatera est triangula*), is a synthetic proposition. For although, if I think three straight lines as enclosing a space, it is impossible that three angles should not simultaneously be formed thereby, I still, in this concept of the three-sided, by no means think the inclination of these sides to one another, i.e., the concept of the angle is not truly thought in it."[122]

The philosopher

> "may reflect on this concept as long as he wants, yet he will never produce anything new. (*CpR* A716; B744)

The mathematician, however, produces new knowledge by construction. In construction, the mathematician is sovereign. Among Kant's Reflections we find the following:

> The mathematician says in his definition: *sic volo, sic iubeo* (this is what I wish, this is what I command)[123]

Maimon could not have known this locus, but in a review of his own *Transcendendental-philosophie* he masterfully captured the air of self-confidence Kant ascribed to the mathematician, and we find in Maimon a completely different view of the nature of this "command":

> The Understanding prescribes the productive imagination a rule to produce a space enclosed by three[124] lines. The imagination obeys and

122. What Real Progress Has Metaphysics Made in Germany Since the Time of Leibniz and Wolff. AA XX, 323, in Theoretical Philosophy After 1781, eds. H. Allison, P. Heath, trans. P. Heath, Cambridge: Cambridge UP.

Maimon could not have known this essay, but Kant gives the same example also in CrV B 621-622.

123. "Der Mathematicus in seiner Definition sagt: sic volo, sic iubeo" (Reflexion 2930, AA XVI, 579)

124. In the text it says: two. When the text was reprinted in the *Philosophisches Wörterbuch* the mistake was corrected. The mistake may have a meaning, though. The "biangle" is Kant's

> constructs the triangle, but lo and behold! three angles, which the understanding did not at all demand, impose themselves. Now the understanding suddenly becomes clever since it learned the connection between three sides and three angles hitherto unknown to it, but the reason of which remains unknown to it. Hence it makes a virtue of necessity, puts on a imperious expression and says: A triangle must have three angles! - as if it were here the legislator whereas in fact it must obey an unknown legislator."[125]

The uniqueness of Maimon's position shows if we compare it to Mendelssohn's (or Wolff) on the one hand, to Kant's on the other. Mendelssohn believed that the proposition "A three-sided figure has three angles" is analytic, its negation a contradiction.[126] This is Mendelssohn's understanding of Leibniz' view that all properties of a substance are virtually contained in its concept as the properties of a circle are contained in its definition.[127] Kant suggested that it is a synthetic judgment a priori, a judgment based on constructing a concept in intuition. But in Maimon's view, the validity of this judgment is at first not at all different from a common empirical judgment based on perception because we have no insight into the reason of the connection (Grund der Verknüpfung) between the sides and angles of the trian-

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example for a concept that does not contain a contradiction and is yet impossible. Leibniz and Maimon use the Decahedron. See *CpR* A220/B267. When Maimon wrote the text, he must have thought of Kant's example for the surplus of intuition. However, what for Kant is the great advantage of intuition, is for Maimon a testimony to the weakness of human understanding.

125. Antwort, GW III, 198-199, Cf. also GW IV, 449-450.

The same criticism applies also to Mendelssohn who gives this example for properties following with "absolute necessity" from the definition. Mendelssohn also considers the possibility of reversing the order of the "*differentia*" and a *segula* using such properties in the definition itself and inferring those now serving as defining properties.

126. Bi'ur Milot ha-Haigyyon, JubA XIV (Breslau 1938), pp. 44, 65, 69, 95. Bendavid shared this view: "Essence and properties are here one and the same." (*Versuch*, XXVII) As an example of "logical truth", Wolff once gave the proposition "Triangulum habet tres angulos" (Logic, Part II, Cap. I, § 505, quoted in German translation in Maimon, GW I, 600).

127. All things are contained virtually in Alexander's concept, as “the properties of the circle are contained in its essence (nature).” (Loemker, 310; (Disc. 13) Mugnai, Leibniz on Individuation, p. 46. On formal and virtual identity Cf. Kauppi: Über die Leibnizsche Logik, Helsinki 1960, pp. 71-76.

gle. So much is clear: Kant did not show that there is a connection between the fact that we construct the triangle and our knowledge that a trilateral figure's has three angles. The property is at first known by looking at the completed triangle whether we constructed it or not. It is purely perceptual knowledge. Looking at the object or imagining it we learn the fact that a three-sided figure has three angles, but not the reason for this fact. True, we feel that this connection is necessary, perhaps not less than if its negation were a contradiction, but it is nevertheless merely subjective necessity imposed on us and not objective necessity established by the understanding. Nevertheless, in the *degree* of certainty, we may approach objective necessity ever closer. (GW IV, 450; Antwort GW III, 198, 199, 200).

Moreover, we can also gain further insight into the reason of the connection between the subject (triangle) and the predicate (having three angles) by conceptual analysis. The judgment thus remains analytic, but it nevertheless enlarges our knowledge. Maimon distinguishes between analytic judgments based on identiy (AB ⊃ B) and analytic judgments which are based not on the concept of the subject but on the subject itself. The latter are analytic and yet informative. In this context he names the example of the triangle having three angles, but he does not elaborate it. I suggest the following argument to illustrate Maimon's reasoning.

The proposition is first known by intuition, not by reason and therefore not objectively. However, the proposition is also not overtly analytic: the definition of the triangle ("Rectilinear figures are those which are contained by straight lines, trilateral figures being those contained by three" *Elements* I, def. 19) does not contain the predicate "containing three angles". We now reason thus: in order to form a closed figure out of three lines these must meet in three points. This is so because each line must meet with another at one of its ends; if they do not meet, the figure is not closed; if they meet at more than one end, the lines coincide (*Elements* I, postulates 1 and 2; hence they must meet at one end.) Three segments have six ends. Therefore, the sides of the triangle must meet at three points. If we further consider the definition of an angle ("A plane angle is the inclination to one another of two lines in a plane which meet one another and do not lie in a straight line" *Elements* I, def. 8), we may infer that a triangle (a trilateral closed figure) necessarily has three angles. This is conceptual, analytic thought that is independent of intuition and yet enlarges our knowledge.[128]

However, our reasoning presupposed that if the ends of two segments coincide, these segments also coincide. This merely reformulates the principle referred to by Kant that a biangle, or a figure enclosed by two straight lines is impossible. (*CpR* A220/B268; see above # 3.1). Maimon's reasoning is indeed analytic - the conclusion follows from the premises - but the premises are richer than only the *concept* of the subject; they involve postulates known by intuition. Geometry is hence not entirely analytic - its principles involve intuition - but some propositions normally based on intuition may be nevertheless rendered analytic.

We see: For both Kant and Maimon "knowledge" requires necessity. Whereas Kant argued in the *CpR* that it is intuition cum understanding that provides synthetic-apodictic knowledge in geometry, Maimon used the very same example to say that certain knowledge is knowledge of the understanding which is not synthetic, whereas intuition provides synthetic knowledge which is not certain, but merely "belief". Maimon's alleged "system of coalition" was in fact a dualism of hope and fear, of hope to vindicate Rationalism and fear of discovering that Man is after all no more than a beast. It was not meaningless that Maimon gave his *Transcendentalphilosophie* the moto: "Dextrum Scylla latus, laevum implacata Charybdis Obsidet."[129]

4.3. *Hypothetical (non-Euclidean) Geometry, Hypothetical Metaphysics*

Maimon draws yet a further consequence from his strict distinction between sensuality and reason, namely that a geometry based on other than the Euclidean axioms would not be less true than the Euclidean, although it would not be "real", i.e. of practical use. The axioms are the "elements" of truth, but not truth itself: Truth consists in "lawful way of thought":

> I am certain that if Euclid has assumed false instead of metaphysically true axioms, he would have left us an opus of no lesser value or quality than that work of his that is still in our possession." (Tr, 149)[130]

128. See Maimon, *Logik*, GW V, 28-31, and GW VI, 174-175.

129. Tr, Title page. Virgilius, Aeneis, Book III, Verses 420-421.

130. See also Tr, 399-406. Maimon expresses the same idea some years later with the distinction between "logical" and "metaphysical" truth. logical truth refers to the consistency of thoughts

It is important to realize that this is an immediate consequence of Maimon's main tenet and not merely a prophetic anticipation of non-Euclidean geometry. Such prophetic anticipations charm posterity[131], but they are of philosophical value only when they are based on a reflection on the nature of geometrical reasoning itself. Moreover, the attempt to "credit" Maimon with an anticipation of non-Euclidean geometry, is flatly belied by his "Concluding Remark" in which he defends the view that the "axiom of parallels" is a synthetic a priori judgment. (See below # 5; This essay of Maimon has been overlooked by scholars until now.) Maimon's argument follows here from severing understanding from sensibility. In other contexts, Maimon explicitly maintains that mathematics is not hypothetical![132] It is even more important to see that Maimon could therefore also draw general conclusions from this observation and apply them not only to mathematics but also to philosophy. Whatever is "given" to either empirical or a priori intuition belongs to sensibility, hence also the proposition that the straight line is the shortest between two points or the axiom of parallels. The understanding draws conclusions according to the laws of logic. If correctly reasoned, the inference conserves the truth-value of the premises and the truth of reasoning is independent of that of the premises. Since Maimon maintains that only reason (inference) establishes objective necessity (truth), and since the axioms from which we proceed are not apodictic - and therefore also not mathematics as a whole,[133] he also maintains that geometry with different axioms (the domain of intuition) would not be less true than Euclidean geometry, provided that the conclusions are correctly inferred (this is the domain of reason).

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with each other, metaphysical thought refers to the consistency between thoughts and objects. Philosophisches Wörterbuch, "Wahrheit", GW III, 182. Maimon repeats his words that the value of geometry would remain undiminished ("because of its theoretical value") irrespective of the truth of its axioms. This time he speaks of "false" (falsche) and "true" (wahre) axioms. GW IV, 240.

131. E.g. Buzaglo, 52-53; Buzaglo is even carried away to the claim that Maimon was "was aware he was on the brink of discovering a different mathematics" (53) and "innovatively advanced the possibility of mathematically fertile non-Euclidean geometries well before Gauss." (37) He makes these claims although he quotes Maimon's words that the Euclidean axioms are "metaphysically" true, i.e. adequate to the physical world. He also does not know Maimon's proof of Euclid's fifth postulate. See #5.1.4.

132. "Wie aber die reine Mathematik hypothetisch seyn soll, ist mir unbegreiflich;" GW V, 395, 398.

133. Tr 184-185. See the discussion in Kauferstein, 206.

In Tr, in *Givat Hammore* and elsewhere Maimon discusses the same example.[134] It is the same proposition which Kant used to show that intuition was indispensable in geometry. Since Maimon draws exactly the opposite conclusion, there can be little doubt that he chose the same example in order to drive the point home. Kant used this proposition (Euclid, *Elements* III, 20) in his "doctrine of method" to argue that the mathematician

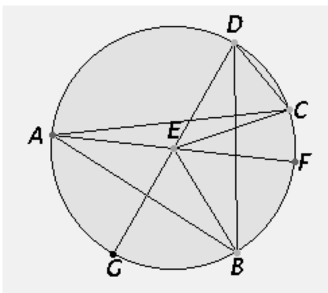
> "arrives through a chain of inferences that is always guided by intuition ... at a fully illuminating (völlig einleuchtenden) and at the same time general solution of the question. (*CpR* A716-717; B744-745)

whereas the philosopher

> "may reflect on this concept as long as he wants, yet he will never produce anything new. (*CpR* A716; B744)

It is conspicuous that Maimon confronts Kant with the opposite view: only the inferences of reason, the domain of the philosopher, are "fully illuminating" (*völlig einleuchtend*), whereas the "new" knowledge arrived at by intuition is merely subjectively true or hypothetical.

> proposition III, 20 in Euclid's *Elements* reads:
>
> In a circle the angle at the centre is double of the angle at the circumference, when the angles have the same circumference as base.



In the proof Euclid uses two propositions proved earlier: I, 5 and I, 32. The latter states:

134. In GM Maimon refers to his discussion in Tr, 148-150. See GM, 135-136, commentary on *Guide* I, 73. See also "Propädeutik zu einer neuen Theorie des Denkens", GW VI, 177.

In any triangle, if one of the sides be produced, the exterior angle is equal to the two interior and opposite angles, and the three interior angles of the triangle are equal to two right angles.

Maimon presents the Euclidean proof of III, 20 and then proves it again, this time altering the conclusion of I, 32 which here serves a a premise such that the exterior angle is supposed to be equal to one and a half times (instead of twice) the sum of the interior opposed angle. It then follows that the angle at the center of the circle is equal to three times the angle at the circumference (instead of twice).

The latter proof is identical to the first and therefore just as valid, although its premise and therefore also its conclusion are different and wrong. Put differently, an inference has the form "if ... then" and is valid irrespective of whether the premise and the conclusion are true or not:

> You clearly see that although the premise assumed true is false and also the consequence is wrong, nevertheless the connection between them is true and conforms to the general laws of thought (GM 136)

And it is also in the context of the discussion of this theorem and its proof that Maimon suggests that a geometry built on other axioms than the Euclidean would be no less true than the Euclidean. (Tr, 148-150)

Now, this approach is endorsed by Maimon not only in respect to geometry but also to metaphysics and to all human thought in general. Maimon suggests that all knowledge is hypothetical since its foundations are always uncertain and the derivations certain. Philosophy, interested in the foundations, may not therefore copy the method of mathematics, i.e. assume axioms and proceed from these.[135] Maimon was presumably the only philosopher who hithero referred to his philosophy as hypothetical. True, he imputed this position to Kant, but Kant

135. "Die so sehr angepriesene mathematische Methode hat, beim genauen Lichte betrachtet, keinesweges den sonderlichen Nutzen, den man sich von ihr verspricht; weil sie so gut zum Fortschritte von Irrthum zu Irrthum, als von Wahrheit zu Wahrheit, den Weg bahnet. Nicht die mathematische Methode also, sondern die Entwickelung der Principien der menschlichen Erkenntniß, aus dem Verfahren des Verstandes und der Vernunft, bei Bildung der mathematischen Begriffe und ihrer Beziehung auf einander, kann diesen Nutzen leisten" (Tr, 285).

would certainly not have been happy with this interpretation[136].

Now, be this as it may, Maimon's critique of Kant is straightforward and clear. Whereas Kant argued in the *CpR* that it is intuition cum understanding that provides synthetic-apodictic knowledge in geometry, Maimon used the very same example to say that the understanding provides certain knowledge which is not synthetic, whereas intuition provides synthetic knowledge which is opaque to reason and not certain. And this position, the core of Maimon's philosophy, is the source of his consideration of the possibility of non-Euclidean geometry: What is given by intuition could be different, what is produced by the understanding is necessary and unique

5. *Maimon's "Concluding Remark": The Nature of Synthetic Judgments a priori and the Axiom of Parallels*

Maimon latest thoughts on geometry are documented in a ten pages essay on the foundations of geometry, entitled "Concluding remark" (*Schlußanmerkung*) and placed at the end of a series of endnotes to the text of his last book, *Kritische Unterschungen* (1797) (GW VII, 362-372). Presumably because of this location and the misguiding title or rather the absence of a title, the essay has hitherto escaped the attention of scholars. The essay is independent of the book, as Maimon himself remarked. Maimon presented here a thesis of which he says that it is "true, new and important," namely that primary, genuine geometrical propositions are logically convertible without change of quantifier and that this follows from their being synthetic judgments a priori. (GW VII, 362)[137] Geometrical propositions that cannot be (concep-

136. Maimon interprets Kant's philosophy, as a "hypothesis" to explain the "fact" of knowledge (Erfahrung), i.e. of synthetic judgments a priori in mathematics and in the fundamentals of physics. Here and elsewhere Maimon, the "critical Skeptic" doubts this "fact"."Herr Kant legt seinem kritischen System Erfahrung, als unbezweifeltes Faktum, zum Grunde, woraus er hypothetisch die Realität der Grundbegriffe und Sätze a priori beweist. Nun hat aber der kritische Skeptiker allerdings Recht, das Faktum selbst (daß wir Erfahrungssätze haben, die objektive Nothwendigkeit und Allgemeingültigkeit ausdrücken) in Zweifel zu ziehn, und folglich auch die darauf gegründete Realität gedachter Prinzipien selbst." (GW III, 420) (See also GW III, 458-459) Maimon imputes Kant this interpretation: "... meiner innigsten Ueberzeugung nach, [hat] Kant nie im Sinne gehabt, duch seyn System die Skeptiker zu überführen." (III, 429)

137. All these statements are included in Maimon's letter to Ernst Gottfried Fischer (1754-1831), July 1, 1797. The Waller Manuscript Collection, Ms de-03559, Uppsala University Library. As he did often himself and also many of his contemporaries, Maimon uses "mathematics" when he actually means "geometry." I am grateful to Florian Ehrensperger for bringing this letter to my attention.

tually) *inferred* from others but have to be proven geometrically (and are, therefore, *genuinely primary*) are synthetic a priori judgments. Among these are the proposition that "The straight line is the shortest between two points", postulate 5, i.e. the so-called axiom of parallels, and those propositions that cannot be proven conceptually but require construction. Nota bene that not all the propositions actually proven geometrically by Euclid are such that they *cannot* be proven conceptually. *Elements* I, 5 is such a genuine geometrical proposition, but *Elements* I, 8 which Euclid also proves can be inferred from a more general one (which is missing from *Elements*). (GW VII, 364)

It thus seems that Maimon finally adopted Kant's position. This however applies rather to the terminology than to the content. Kant defined synthetic judgments in contradistinction to analytical judgments, in which the predicate is "covertly" contained in the subject term and, therefore, partially identical with it. Analysis of the subject term reveals the component identical with the predicate. In synthetic judgments, the predicate is "entirely outside" the subject term and yet connected with it. (*CpR* B 10-12) Maimon does not name such judgments "analytical judgments", but rather "judgments of identity" and excludes them from further discussion. In his "*Logik*" he maintained that thinking identities is not at all thought, in the *Kritische Untersuchungen* to which our text is attached he says that judgments of identity are "barren" (unfruchtbar). (GW VII, 134)

In Maimon, analytical judgments are those that can be inferred conceptually, although in a mediated way, be it that they are inferred from a more general case, or that the subject of the proposition can be *reduced* (not analyzed!) to another subject of which the proposition in question is already established.[138] In contradistinction to these, synthetic judgments are those in which the property is immediately predicated of the subject (as "shortest" of "straight line").

The announcement of the new and important discovery and Maimon's new discussions in this essay should not obliterate the astounding continuity in his thought. To Maimon, synthetic judgments a priori are those that resist reduction to analytic judgments and, therefore,

Ehrensperger prepares the publication of this letter.

138. On Maimon's "Reduktion", see (GW VII, 418-419). The example given there is the reduction of an isosceles triangle to two triangles of which two sides and the enclosed angle are equal.

remain not understood. An immediate predication "A is B" ("The straight line is the shortest between two points") is synthetic a priori as it is also opaque to reason. Furthermore, the criterion of convertibility without change of quantifier is exactly the same as the one that Ibn Tibbon formulated for segula, i.e a *proprium*. (See above § 2.7). And indeed, the example of synthetic judgments a priori Maimon considered here is again "The straight line is the shortest between two points" (GW VII, 364; see also VII, 133ff) which in his commentary on Maimonides' *Guide of the Perplexed* he called *segula* (*proprium*). Maimon's "important discovery" concerning synthetic judgments a priori is hence first that Kant did not improve on Aristotle's idion (*proprium, segula*), second that all genuine geometrical propositions are such synthetic judgments a priori. If a proposition *must* be proven geometrically and cannot be inferred logically, then it is synthetic; and if it *can* be proved geometrically, it is a priori. This also holds in the case of the straight line. This proposition, too, is convertible without change of quantifier and is therefore synthetic a priori:

All straight lines are shortest between two points

All shortest lines between two points are straight.

The same holds for the properties trilateral and triangular figures (GW VI, 174-175)

Now, the first conclusion from these considerations is that synthetic judgments a priori are *sui generis* and opaque to reason. *Predicatum inest subjecto* does not apply to them, nor are they synthesized by the understanding. If they were synthesized by the understanding, the constitutive property and the *proprium* would have been connected in the *concept*, but they are connected only in the *object* or in its *construction*. The connection is rather imposed on us as empirical facts are. In short: For the finite understanding (!) synthetic judgments a priori exist and they resist elucidation, "synthesis" remains an empty word.

However, not all judgments that look like synthetic judgments a priori are genuine. Since synthetic judgments a priori cannot be elucidated, the task of philosophical analysis is to reduce their number to a minimum and thus rationalize geometry as much as possible or, to put this succinctly, to elaborate the program of Dogmatic Rationalism. This is what the "Concluding Remark" exemplifies for some crucial examples from geometry.

5.1. *Primary and Derivative Geometric Propositions*

Maimon claims that his logical and transcendental arguments suffice to establish his criterion of synthetic judgments a priori, but he nevertheless continues to discuss some geometrical cases that could be adduced as counter-examples. These discussions are of special interest because they pertain inter alia to the notorious use of "congruence" (*Elements* I, 4) and to the "axioms of Parallels" (*Elements* I, postulate 5; in the then common numbering in German: "the eleventh principle"). Maimon's consideration is based on the order of determinability or the converse order of implication: Congruence implies equality (but not vice versa), equality implies proportionality (but not vice versa). If the subject implies the predicate and the judgment is not a mere tautology, then it cannot be converted without change of quantifier. This gives us an additional, negative criterion of synthetic a priori judgments in geometry: propositions that are not logically derived from others and must, therefore, be proven geometrically, are not analytic and yet a priori: they are synthetic a priori![139] Therefore all genuine "primary" geometrical propositions are synthetic a priori. This does not mean that all propositions in Euclid's *Elements* are genuine primary geometrical propositions. It may be that they are covert corollaries of such primary propositions.

Maimon distinguishes three kinds of propositions that seem to be synthetic a priori, but in fact are not:

- (a) When the proposition is not primary but a corollary of a synthetic judgment a priori in *Elements*

- (b) When the proposition is not primary but a corollary of synthetic judgments a priori that is not explicitly stated in *Elements*

- (c) When the conversion itself is mistaken.

5.1.1. *Genuine Primary Geometrical Propositions*

Consider first genuine primary geometrical propositions which are synthetic judgments

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139. This idea shows already earlier in Maimon's writings but is not systematically developed: "Der Satz z.B. Wenn sich zwei gerade Linien einander schneiden, so sind die Schneidewinkel einander gleich, ist ein bloß mathematischer Satz, indem die Gleichheit der Schneidewinkel nicht durch Einerleiheit des Begriffs, sondern bloß durch Konstruktion dargethan werden kann. Dahingegen dieser Satz: der Inhalt des Zirkels ist dem Rektangel aus dem halben Diameter in die halbe Peripherie gleich, ist ein Produkt des Philosophirens über die Mathematik, indem diese Gleicheit aus der Einerleiheit von dem Begriff des Zirkels mit dem Begriff eines Polygons von unendlich vielen Seiten, dargethan wird." (GW IV, 243-244)

a priori. Maimon mentions not only the proposition that the straight line is also the shortest between two points and that a trilateral figure is also a triangular, but also *Elements* I,5 (GW VII 362 and 367-368). The first part of the proposition reads:

> In isosceles triangles the angles at the base equal one another

And indeed, the converse of this proposition is also true and Maimon's criterion of synthetic judgments a priori is hence satisfied. In fact, it is the next proposition, I, 6:

> If in a triangle two angles equal one another, then the sides opposite the equal angles also equal one another.

Maimon draws the conclusion. Since I, 5 is a primary geometrical proposition which cannot be logically inferred from another, it is a synthetic Judgment a priori and synthetic judgments are convertible:

> Having proven propostion I, 5, Euclid could therefore be certain in advance that also I, 6 could be proven. (GW VII, 367-368).

Maimon seems to commit a blatant mistake here. It is not enough to prove that in isosceles triangles the angles at the base equal one another in order to convert the proposition. We also have to prove the inverse proposition, namely that (all) triangles that are not isosceles do not have equal angles at the base. This proposition is the conclusion of *Elements* I, 8 which Maimon discusses in this text in detail. But Maimon does not refer to I, 8 in order to convert I, 5 to I, 6. Without this additional condition we can infer by conversion only that in *some* triangles, the sides opposite respective equal angles are also equal.

We see here what seems to be a double mistake of Maimon. He first errs in the elementary conversion rules of logic (three years after the publication of his book on logic!), and second, he does not use the propositions at hand to geometrically prove the proposition he wishes to establish. I will discuss Maimon's notion of conversion in # 5.1.3 below.

Now, Maimon's criterion of synthetic judgments a priori immediately proves productive in comparing I, 5 and I,8. I, 8 states:

> If two triangles have the two sides equal to two sides respectively, and also have the base equal to the base, then they also have the angles equal which are contained by the equal straight lines.

The similarity to I, 5 could mislead us into believing that also the converse of I, 8 is true. Yet it is not. The correct conversion is that *some* triangles that have all angles equal to one another have also the respective sides equal. We can also say that if the corresponding angles are equal to one another, the triangles are similar (not equal!), i.e. that the sides are proportionate to each other. This is an equality of ratios, not of magnitudes. Equality of magnitudes implies the equality of their ratios but not conversely. Therefore, I, 8 is not a synthetic judgment a priori or not proven geometrically, but rather a corollary of a primary proposition:

> If the sides of a triangle are proportionate to the respective sides of another, then etc. [they also have the angles equal which are contained by the proportionate straight lines]. (GW VII, 364)

From this proposition I, 8 trivially follows.[140] Maimon says that I,8 exemplifies cases of "analytic propositions" which are derived from original "primary" synthetic propositions proven geometrically. However, Maimon does not prove his adaptation of I, 8 independently of the Euclidean I, 8, nor does he elaborate an alternative structure of *Elements*. In Euclid himself, a proposition like Maimon's adaptation of I, 8 is indeed proven. It is *Elements* VI, 5:

> "If two triangles have their sides proportional, the triangles will be equiangular and will have those angles equal which the corresponding sides subtend."[141]

The proof, however, involves *Elements* I, 18! The lesson is important. Maimon argues that equality of magnitudes implies equality of ratios but not vice versa. He does not argue that the "primary" geometrical proposition can be proven geometrically. In fact, many propositions use "equality" in their wording but can be proven only because they apply the stronger equality "congruence". This is the case already in I, 5 discussed above: The data given use "equality", but in fact congruence is used in the proof from which equality of magnitudes trivially follows. This is so because the proof of I, 5 proceeds from the given data

140. (AB:DE :: AG:DF :: BC:EF ⇒ B=E ∧ C=F ∧ A=D) ⇒ (AB = DE ∧ AG = DF ∧ BC=EF ⇒ B=E ∧ C=F ∧ A=D) Maimon abbreviates: A = B ∧ C = D ⇒ A : B = C : D.

Maimon brings I, 18 also as an example of a "complex" proposition which cannot be directly converted.

141. Heath, III, 202-204.

that two sides of the isosceles triangle are equal to one another and the angle contained by them is equal to itself. On the basis of these equalities and an additional construction, Euclid proves that two triangles in the diagram are "equal" to one another. This is Euclid's wording. However, side-angle-side equality establishes congruence. Euclid hence proves that these pairs of triangles *coincide* with each other and concludes that the relevant magnitudes are also equal. This will be discussed below when Maimon's analysis of I, 4 will be considered. However, note that Maimon is interested in the reverse side of what Kant focused on. Kant concentrated on the role of intuition in geometry, Maimon looks for the logical structure (of pure understanding) irrespective of whether on this basis geometry can prove its theorems. Genuine primary geometrical propositions appear where logic ends, and Maimon attempts to restrict their domain as much as possible.

5.1.2. *Pseudo Primary Geometrical Propositions*

Proposition I, 5 is a genuine primary geometrical proposition and a synthetic judgment a priori. Proposition I, 8 was interpreted as a pseudo primary geometrical proposition which is in fact a corollary of a primary proposition. To prove I, 8 Euclid uses I, 4 and this in turn is proven on the basis of Common Notion 4:

"Things which coincide with one another equal one another."

Now, Common Notion 4 and proposition I, 4 in which it is used were considered problematic by many commentators because "coincidence" seems to involve the motion of geometrical figures and therefore be empirical (See Heath I, 224-231, 248-250). This is not Maimon's concern here.[142] Maimon rather shows that and why this Common Notion is not a synthetic judgment a priori and therefore not a primary geometrical proposition, but an implied corollary. Evidently, the proposition cannot be converted without change of quantifier: not all things which are equal to one another also coincide with one another. Rather, only some such things do coincide. Hence, the proposition must be analytic. Indeed, Maimon suggests elucidating it as follows.

Magnitudes that have the same figure, position and quantity have the

142. In a different context, Maimon argued that *Elements* I,4 does not involve motion. See GW VII, 190-191.

same quantity. (GW VII, 366)

Predicatum inest subjecto. The proposition cannot be converted without change of quantifier because "coincidence" is more specific than "equality".[143] "Some" but not "all" equal magnitudes have also the same figure and position.

In fact, this is not only valid for Common Notion 4 but also for proposition I, 4:

> If two triangles have two sides equal to two sides respectively, and have the angles contained by the equal straight lines equal, then they also have the base equal to the base, the triangle equals the triangle, and the remaining angles equal the remaining angles respectively, namely those opposite the equal sides.

Now the proof notoriously "applies" one triangle to the other. To avoid the empirical and mechanical ring involved in introducing motion into geometry, some commentators suggested including proposition I,4 among the axioms. Is it hence a synthetic judgment a priori? Applying the criterion of convertibility, Maimon reaches a negative conclusion. The proof of I, 4 uses the equality of side-angle-side. This equality establishes not only equality but congruence. In order to see whether the proposition can be proven only by congruence, consider the case in which two sides and an angle can be constructed equal to their counterparts in another triangle but not coinciding with them. To do this, let the angles in question not be contained by the two pairs of equal sides. Let ABC, DEF be two triangles, A = D, AC = DF and BC = EF. Is here, too, AD = DE?

This "ambiguous case" has two solutions,[144] the conversion is hence not valid. Now, in I, 4 the equality of triangles is proven from the equality of two sides and an angle. In what is

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143. Maimon writes: "weil Decken allgmeiner als Gleichsein ist." (VII, 366) This is an obvious mistake, it should be the other way around, as correctly put some lines above: "Eben so ist das Verhältnis überhaupt allgemeiner als Verhältnis der Gleichheit, (indem es ratio aequalitatis und inequalitis gibt.) (GW VII, 366)

144. Heath formulates the proposition in the following way: "If two triangles have two sides equal to two sides respectively, and if the angles opposite to one pair of equal sides be also equal, then will the angles opposite the other pair of equal sides be either equal or supplementary; and in the former case, the triangles will be equal in all respects." (Heath I, 306) Maimon writes that the proposition will not be generally true "but only under the condition that BC>AC and EF>DF." (GW VII, 368)

this "ambiguous case" different from I,4? As we have seen, the answer is simply that the wording of I, 4 had it that two sides and an angle were equal, but it referred in fact to congruence. Finally, consider the case in which only two respective sides of the triangles are said to be congruent. To construct this, also the angle contained by these sides must be equal, and this conversion of Proposition I, 4 is identical to the original I, 4. In a word: The wording of the proposition alone may sometimes mislead the reader.

In sum, then, Maimon did not change his philosophical position nor his motivation since the first draft of *Transcendentalphilosophie* although his views on geometry changed radically. At first Maimon believed that he could reduce each and every geometrical, seemingly synthetic proposition to an analytic implication. When his attempt to reduce the proposition "The straight line is the shortest between two points" failed, Maimon admitted that there are synthetic proposition that the finite human mind cannot reduce to others. He even considered the possibility that these judgments are merely based on induction. Here, almost at the end of his short career as a philosophic writer, Maimon explicitly acknowledges that all genuinely geometric propositions are synthetic a priori, and he is willing to include in these all propositions which cannot be derived from others, (e.g. I, 5). However, the motivation did not change. Maimon's aim is to clearly distinguish between valid propositions which are imposed on us in intuition and true propositions concerning which we have insight into the reasons of their truth. Moreover, his analysis showed that sometimes the wording of the geometrical proposition misleads us as to its real logical structure: through the diagram in intuition a stronger premise sneaks in, and we actually infer from "congruence" although in words the proposition speaks only of "equality". Only logic (logical analysis, the test of convertibility) can reveal the real structure of the argument.

5.1.3. *Conversion of Primary Geometrical Propositions*

Although Maimon names an extensional criterion for synthetic judgments a priori (convertibility without change of quantifier), his entire thought aims at an intensional interpretation of logic. He gave a justification of his view in his "Propädeutik zu einer neuen Theorie des Denkens" which he attached to his translation of the categories of Aristotle of 1794. In the chapter titled "On Logic as such (überhaupt)", # vi-viii (GW VI, 170-175)[145], Maimon attempted to show that quantified propositions are in fact abbreviated syllogisms of not quantified propositions.

> The classification of judgments according to their quantity has no philosophical origin and is adopted from their usage in common life. In fact these [propositions] are abbreviated syllogisms (Schlüsse) or the conjunction of some judgments without quantity.
>
> For example, this universal affirmative judgment: "A human being is a living being" is the conclusion of the following syllogism of reason:
>
> Human is animal
>
> (Animal is the determinable, humanity one of its possible determinations and human the thus determinated.)
>
> All humans (cajus, Titius etc.) are humans
>
> (Humans are now the determinable, Cajus, Titius etc. the determinated.)
>
> Therefore all humans are animals (GW VI, 170)

Maimon proceeds to elucidate in a similar way universal negative judgments as well as particular positive and negative judgments. The purpose of the elucidation is clear: the intensional interpretation shows that true quantified judgement can be reduced to the form *predicatum inest subjecto*. The major premise in this syllogism is an analytic statement: *animal rationale est animal*, AB is A. The judgment is hence certain (gewiß), and of absolute generality. In contrast, without this justification, universal judgments are merely inductively justified and hence of comparative generality only (GW VI, 171-173).

The order: Cajus, human, animal is the converse of Maimon's Law of Determinability, and the quantified judgment "Some animal is human" is the converse of "All humans are animals". In intensional terms we may say: "Animal can be human" (among other possible determinations), "Human can be Cajus". Moreover, on the level of the concepts the difference shows in the fact that the subject can be thought independently of the predicate, but not vice

145. See also the sixth letter to Aenesidemus, *Logik* (1794), pp. 408-409.

versa: "human" implies "animal", but "animal" does not imply "human".[146] Hence the analysis of a covert analytic judgment shows that *predicatum inest subjecto* and that the judgment is necessary, whereas the analysis of a synthetic proposition like "Animal can be human" shows that the judgment conforms to the transcendental structure "subject-predicate" and, therefore, is a well-formed judgment, but not that it is true.

The judgment "The straight line is the shortest between two points" belongs to neither of these two types of judgments. It is not analytic because "shortest" is not implied by "straight" nor identical with it, and it seems not to be synthetic because it does not conform to the Law of Determinability. A predication conforming the to Law of Determinability constitutes a new subject with new consequences which follow neither from the subject nor from the predicate. But the predication of "straight" respectively "shortest" to "line" do not form two new objects, each with its peculiar consequences, but one object only. "Straight" and "shortest" are coordinated, not subordinated, and yet they do not exclude (e.g. like colors) each other but, on the contrary, are "inseparable". In quantified terms, this shows in that the converse of "All straight lines are shortest between two points" is not "*some* shortest lines between two points are straight", but "*All* shortest lines between two points are straight". It seems as if they are "correlatives", like "cause" and "effect" that imply each other although they are not identical and neither is contained in the other. "A is the cause of B" implies "B is the effect of A". But this is not so: "straight" and "shortest" are not "of" the other (as cause and effect are), they are not defined by each other, and each can be thought independently of the other. (GW II, 37; GW VI, 78)

Maimon finally suggests that synthetic judgments a priori are "reciprocal judgments". In the "Propädeutik" his example is that a trilateral figure also has three angles. Each of the properties can be predicated of the same subject and form a determined subject independently of the other and yet:

> These two predicates are necessarily connected with each other such that as soon as the subject (figure) is connected to one of its possible

146. In "Propädeutik" Maimon says that the concept can be an "object of consciousness" (Gegenstand des Bewußtseins"; GW VI, 174), in the *Transcendentalphilosophie* that the subject can be thought independently of its determiantions (Tr, 84, 377-378).

> predicates (three sides) and forms a new subject (trilateral figure), also the other predicate (three angles) must be attributed to it and vice versa. (GW VI, 174)

In a word: triangularity is a *proprium* of the trilateral figure (and vice versa). We can now return to *Elements* I, 5 and I, 6 discussed above and understand why Maimon maintains that I, 6 follows from the conversion of I, 5 without change of quantifier. If in a true predication subject and predicate are co-extensional, then the predicate is a *proprium* and the judgment a synthetic judgment a priori. Or the other way around: If a predication cannot be inferred but can be proven a priori geometrically, then the proposition is synthetic a priori. Since in synthetic judgments a priori the predicate is coextensional with the subject, they can be converted without change of quantifier.

In I, 5 we prove geometrically a priori that the angles at the base of an isosceles triangle are equal. The equality of angles is not contained in the concept of equal sides of a triangle. The proposition is hence synthetic a priori. The equality of the angles at the base is a *proprium* or a *segula* of the isosceles triangle and proposition I, 5 may be converted without change of quantifier and yields I, 6.

> Having proven the fifth proposition of book I, Euclid was rightly certain that he would also be able to prove the sixth. For truly, a logical or transcendental proof is not worse than a geometrical, but even has an advantage over it thanks to its universality. - This does not dispense the geometer from looking for a geometrical proof, but not in order to assure himself of the truth [of the proposition], but in order to establish and expand [this] science on its own ground. (GW VII, 368)

It was typical of Maimon not to say a word in the present context on his intensional interpretation of inference or its application to the theorems considered here. He thus seemed to commit trivial mistakes where in fact he had a profound justification.

5.1.4. *The Axiom of Parallels: A synthetic Judgment a priori*

In the last sections of his "Schlußanmerkung" Maimon's applies his criterion of convertibility to a contemporary discussion over the axiom of parallels. Maimon addresses a controversy between two renowned mathematicians Wenceslav Johann Gustav Karstens (1732-1787) and

Carl Friedrich Hindenburg (1746-1808). Hindenburg published in 1781 a "New System of Parallels" and Karstens criticized it in a treatise titled "Of the Parallels and the New Attempts to complement their Theory".[147]

In Karstens' view, Euclid proved in a satisfactory way the following proposition:

> If a straight line falling on two straight lines makes with them two interior angles, the sum of which equals two right angles, then the straight lines are parallel to one another. (This is part of *Elments* I, 28) (Karstens #4, p. 118)

The conversion of this proposition

> Parallel lines make with every straight line cutting both of them two interior angles, the sum of which equals two right angles. (Karstens, # 5, p. 118; GW VII, 370)

What is the correct quantifier of this latter proposition: Is it "*All* Parallel lines" or "*Some* pararallel lines"? The logical rules of conversion, says Karstens, allow only to infer from the universal "All A is B" the particular "Some B is A". If we wanted to infer "All B is A", we had first to establish that: "All that is not A is not B" (Karstens #5, p. 119). Applied to the relevant case here:

> If a straight line falling on two straight lines makes with them two interior angles, the sum of which *does not* equal two right angles, then these straight lines *are not* parallel to one another. (Karstens # 6, p. 120)

But Euclid could not prove this latter proposition and therefore also not the proposition that all lines that do not make with the transversal interior angles equal to two right angles are not parallel. He therefore had to introduce this proposition as a postulate (i.e. the axiom of parallels):

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147. Hindenburg's "Neues System der Parallellinien" was published in Leipziger Magazin zur Mathematik, Naturkunde und Oeconomie, 2 (1781). Karstens' critique, "Von den Prallellinien und den neuen Bemühungen, die Theorie davon zu ergänzen" is contained in his Mathematische Abhandlungen, Halle (Rengersche Buchhandlung) 1786.

> That, if a straight line falling on two straight lines makes the interior angles on the same side less than two right angles, the two straight lines, if produced indefinitely, meet on that side on which are the angles less than the two right angles. (*Elements* I, Postulate 5)

Such lines are parallel:

> Parallel straight lines are straight lines which, being in the same plane and being produced indefinitely in both directions, do not meet one another in either direction. (*Elements*, I, Definition 23)

Maimon attacks Karstens' argument and maintains that the inference of the universal proposition equivalent to postulate 5 is valid also without first proving that "All that is not A is not B". This claim, of course, blatantly violates the basic rules of standard logical conversion, but we know already that Maimon does not apply the rules of extensional logic. He rather reasons as follows: If a judgment is a true synthetic judgment a priori then it must be proven geometrically and it predicates a *proprium* of a subject. The *proprium* and the subject are coextensional and may, therefore, exchange places in the judgment: "The straight line is the shortest between two points" implies here "The shortest line between two points is straight." Extensionally expressed, the propositions can be converted without change of quantifier. Therefore, if *Elements* I, 28 is true as Karstens concedes, then also its conversion (without change of quantifier) is true and therefore postulate 5. Maimon concludes:

> The first of these propositions that Euclid proves [i.e. *Elements* I, 28] is a synthetic proposition. It can hence be converted without change [of quantifier] and the second formulated as a universal proposition. From this the notorious eleventh principle can now be inferred according to a legitimate conversion rule known since long time. [148] But since this

148. The first conversion that is not generally valid but only for genuine primary geometric proposition as *Elements* I, 28 yields: "All Parallel lines make with every straight line cutting both of them two interior angles, the sum of which equals two right angles. "

From this we obtain by standard contraposition:

"All non-parallel lines make with a transversal two interior angles the sum of which is not equal to 2R." This proposition is equivalent to postulate 5.

principle [i.e. postulate 5]is thus proved only logically but not geometrically, Euclid (who wished to prove everything geometrically) placed it among the principles. But it is not self-evident! [alluding to an objection of Karstens # 8, p. 121] To this I reply: it need not be. It was proven logically, and therefore its truth cannot anymore be doubted, even if it should not be possible to prove it also geometrically. (GW VII, 371)

But why did Maimon believe that proposition I, 28 was true? Maimon could have also thought that this genuine primary geometrical proposition that cannot be proved by logic is a deception of the intuition as common-sense mistakes concerning the *rota Aristotelis* (#see 3.8) or the asymptotes (see # 4.1). But in fact, notwithstanding the *philosophical* critique of geometry, because of its dependence on intuition, Maimon's notion of true synthetic knowledge was dependent on Euclid's geometry. Geometry was the exemplification of synthetic true knowledge. Maimon's rationalism justified only analytic truth. Truth and real, synthetic knowledge, Truth and real thought fell apart. Geometry could hence either be true - and virtually analytic - or truly synthetic - and merely imposed on us (but not necessarily true).[149]

Maimon distrusted intuition. This distrust could have motivated a suspicion against geometry or a wish to reduce geometry to the understanding. Here, with the proof of the parallel-axiom, Maimon went the latter way. His trust in synthetic certain knowledge and his trust in its reducibility to logic overcame his suspicion of intuition. Instead of upholding his reservation against geometry which builds its fabric on axioms that are opaque to the understanding and merely imposed on intuition, Maimon took the optimistic alternative and accepted synthetic geometric truth as valid although deficient in its form. The proper form is logical inference. And since Maimon was truly convinced of the truth of *Elements*, he anticipated the infinite analysis of synthetic truths and replaced in this case "A is B" with "AB is B". After this replacement, Maimon’s argument follows correctly.

Note that Maimon's mistake in this argument is not that he believed that postulate 5 is

149. "Was mich anbetrift, so eigne ich der demonstrirten mathematischen Erkenntniß objektive Wahrheit (Uebereinstimmung mit ihren Principien), den Axiomen aber blos objektive [should be: subjektive] Nothwendigkeit zu, in so fern sie von keinem besondern Umstande des denkenden Subjekts abhängen." GW III 184-185

true. Maimon had two pairs of deeply entrenched opposing beliefs: the belief that *Elments* are true opposed the distrust in intuition and the belief that all truths are virtually analytic opposed the belief that truth was analytic for the infinite understanding only. The wish to prove the fifth postulate and to show that at least the elementary propositions of *Elements* can be proven true *strictu sensu* overpowered the distrust in intuition. Here "The wish was father of that thought" and allowed Maimon to accept the synthetic predication as a true predication of a *proprium* and then replace this predication with an analytic proposition.

Neither Maimon's motivation nor his philosophical position changed since his *Transcendentalphilosophie*. There, too, Maimon attempted to reduce a synthetic judgment a priori to an analytic implication, namely that the straight line is the shortest between two points. His failure then led him to adopt in addition to his dogmatic rationalism his empirical skepticism. He never gave up these two alternatives. But in the "Concluding Remark", Maimon was once more carried away by the hope that he could prove a synthetic judgment a priori, and he again committed a mistake. Since this is the last book he published, we do not know whether in this case, too, he later recognized his mistake.

And yet, Maimon's mistake is quite irrelevant to his philosophical position. The "axiom" of parallels may be successfully reduced to an analytic inference or not. If it is an analytic inference, it is true but not synthetic. And if it is synthetic, then it is a primary geometrical proposition and cannot be called true. Synthetic axioms are merely imposed on our intuition (#4.3).[150] We have to accept them, but we have no insight into their truth. "Synthetic a priori" would have been the peak of human knowledge if we indeed synthesized the judgment, if the understanding constructed and understood the bond between subject and predicate, but this is not the case here: We think a trilateral figure and do not, therefore, think of three angles. However, when we construct the trilateral figure in intuition, three angles also impose themselves on us. Kant was tempted to believe that this is a synthetic work of the understanding: " sic volo, sic iubeo" (this is what I wish, this is what I command), Maimon suggested that this is self-deception. Actually, we make "virtue of necessity", we put on an "im-

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150. "Was mich anbetrift, so eigne ich der demonstrirten mathematischen Erkenntniß objektive Wahrheit (Uebereinstimmung mit ihren Principien), den Axiomen aber blos objektive [should of course be: subjektive] Nothwendigkeit zu, in so fern sie von keinem besondern Umstande des denkenden Subjekts abhängen." GW III 184-185

perious expression" and say: "A triangle must have three angles! - as if [the understanding] were here the legislator whereas in fact it must obey an unknown legislator." (See # 4.2) In the case of the axiom of parallels, Maimon himself fell prey to the same hubris and convinced himself that he could base it on logic.

6. *Maimon's Notion of Construction and the Nature of a Philosophical System*

Maimon's criticism of Kant's notion and examples of construction in geometry aimed at the core of Kant's suggestion as to how synthetic judgments a priori are possible. And yet, Maimon too stressed the role of construction not less than Kant. Very emphatically, he once compared man's role in construction with God's role in creation:

> "God, as infinite power of representation (Vorstellung), from all eternity, thinks himself as all possible essences, that is, he thinks himself as restricted in every possible way. He does not think as we do [namely], discursively; rather, his thoughts are at one and the same time presentation (Darstellung; complete exhibition). If someone objects that we have no concept of such style of thinking, my answer is: We do in fact have a concept of it, since we partly have this style in our possession. All mathematical concepts are thought by us and at the same time exhibited as real objects through construction a priori. Thus, we are in this respect similar to God." GW IV, 42[151]

The enthusiasm about construction and the severe criticism of Kant supplement each other. This is so because Maimon indeed believed that construction is essential to knowledge, but had a notion of construction entirely different from Kant's. His criticism of the attempts to construct even the straight line and the circle were discussed above. But Maimon does not share Bendavid's view that elementary geometry is more "evident" than higher geometry. Also the objects of higher geometry can be rigorously constructed: An ellipse, a parabola and

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151. Translated in Lachterman (1992), 498-499. See the discussion in Schechter, pp. 44-47.

a hyperbola can be constructed just as well as a circle.[152] In fact, Maimon suggests constructing also the objects of elementary geometry on the basis of higher geometry, namely by means of conic sections. This means that objects are not to be constructed using more elementary, but, on the contrary, from more complex objects, not by composition, but by specification, in fact by "determination" according to the Law of Determinability. With this suggestion, Maimon presents an entirely different and new notion not only of construction, but also of a conceptual "system", philosophy included.

Genuine synthetic knowledge consists in the insight of the connection between the essence (definition) of the object and its *propria*. This applies to a single proposition and to knowledge in general. The "necessity and universality required for science", could be attained, so Maimon believed,

> if we could subordinate all objects of human knowledge to one and the same concept. (IV, 64-65)[153]

Now, the obvious problem of such a program is that we need a principle of determination and further specification of this "one and the same concept" which is not merely explicating what was encapsulated in the concept and, therefore, generates new knowledge. At the very same place Maimon clearly says what he has in mind. He speaks of two definitions of a circle, the definition of "common" geometry and the definition of higher geometry. The definition of a circle in "common" geometry and its construction by motion were discussed above. In higher geometry we have a general definition from which the properties not only of the circle follow but also of all other figures which fall under this general definition, as well as the relations among these.[154] Moreover, we also obtain from this definition rules of construction for the

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152. See Tr 275-276.

153. Here evidently Maimon's conception of the "supreme syntheses" is meant which cannot be discussed in this context.

154. Maimon's wording leaves much to be desired, but the meaning is unequivocal: "Wir haben aus der gemeinen geometrie, einen Begrif von einem Zirkel; woraus wir seine Eigenschaften herleiten. Aus der höhern Geometrie haben wir einen allgemeinen Begrif von einem Zirkel; woruas wir seine eigenschaften herleiten. Jener Begrif ist also nicht präzis genug, indem er auch die eigenthümlichen Merkmale des Zirkels enthält, die zur Herleitung dieser Eigenschaften entbehrlich sind. Durch Vergleichung des Zirkels mit andern Figuren erlangen wir also einen präzisen Begrif von demselben." (IV, 64)

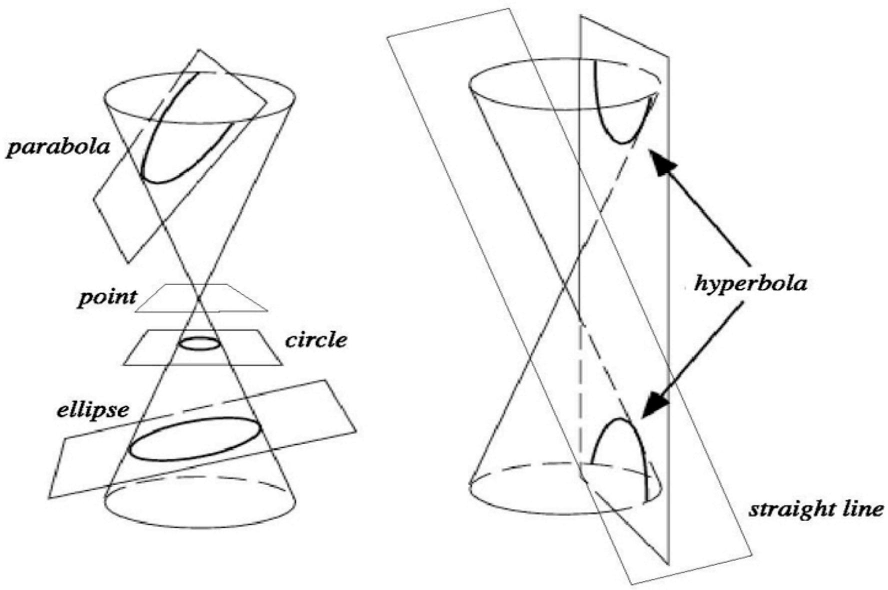
circle (and the other forms) without motion. Maimon means the concept of a conic section :

> Thus in ordinary geometry the circle e.g. is defined as a line all parts of which are equally distant from a certain point (the center). The consequences to be drawn from this concept are only valid for the circle, not also for other curved lines. In higher geometry, the circle is determined as a curved line of the second order by a general equation. The consequences to be drawn from this equation are therefore valid not only for the circle, but for all lines of this order, etc." (GW IV, 612)[155]

Maimon refers here to the construction of a circle as a conic section and he explicitly refers to the algebraic equation: "The circle is determined as a curved line of the second order by a general equation." This is true but insufficient. The circle is *determined* by the equation, but cannot be *constructed* on its basis alone, as Maimon himself insisted. The equation assigns only single *loci geometrici* on the curve but not its continuous outline itself (see above, # 3.7).[156] And it is from the need to construct the cone that Maimon's important insight emerges.

155. Kant, too, once considered the construction of a circle as a conic section. Moreover, he also noted that then a property that Euclid proved for the circle (*Elements* III, 35) can be proved for all conic sections. However, he does not draw from this example consequences concerning mathematics but rather that physics has to conform to the geometrical properties of space as proven in geometry. See Prolegomena, # 38, AA IV, 320-321

156. See e.g. Henk J. M. Bos, Lectures in the History of Mathematics, The American Mathematical Society, 1993, lectures 2 and 3, pp. 23-58. See also Henk J.M. Bos, "On the Interprettion of Exactness," in: Czermak, Johannes [Hrsg.]: Akten des 15. Internationalen Wittgenstein-Symposiums : 16. bis 23. August 1992, Kirchberg am Wechsel (Österreich), Bd. 1: Philosophie der Mathematik, pp. 23-44.



Let us consider a simple case. Consider a circular cone cut by a plane. The boundary curve of the intersection is a conic section. According to the angle of intersection this conic section is an ellipse, a circle, a parabola or a hyperbola, yes even a point, a straight line and intersecting straight lines can be thus produced. If the plane intersects the apex of the cone parallel to its napes it produces a straight line (or intersecting lines), if it intersects the vertex of the cone parallel to its base, it produces a point.

We see here "genetic definitions" or construction rules not merely of the circle but of other geometrical objects which do not introduce motion and by which the circle and each of the other figures is defined as a specific kind of a more general genus (i.e. by a *differentia specifica*). Thus also the relations between these different elements are transparent, and they are also transparent in the algebraic representation. It is important to note that whereas we may continuously alter the angle of the intersecting plane, the construction does not depend on a continuous alteration and, therefore, on motion. In fact, a few distinct sections produce all kinds of the geometrical objects we need. The definition by *genus proximum* and *differentia specifica* is here not imported from Aristotelian logic into mathematics, but produced in mathmatics itself. Not surprisingly, this case beautifully exemplifies Maimon's Law of Determinability. If we were not finite as we are, we could thus construct all geometrical objects beginning only with the most general concept of space.

This is the model of Maimon's process of determination and specification of general concepts. Note that we produce the kinds subsumed under the concept of the genus, we do not unfold the concept and discover the kinds that were encapsulated in it. Here the objects are produced according to specific rules of construction (the intersection of the plane and the cone in specific angles) but not inferred from the concept. The judgments are therefore both synthetic and a priori.

In response to Kant's critique of metaphysics, Eberhard maintains that it is legitimate to cultivate a science without proving the "transcendental validity" of its truths, i.e. their objective validity. As an example from mathematics for this procedure, he names conic sections. Appollonius and his commentators elaborated the theory of conic sections without first showing how the ordinates are to be drawn onto the diameters of these curves, although the "reality of the whole theory" depends on the possibility of drawing these ordinates.[157]

In his answer to Eberhard, Kant insisted that Appollonius "first constructs the concept of a cone, i.e. presents it a priori in intuition" and thus demonstrates its reality.[158] Kant believes that Eberhard understands "construction" to mean "mechanical construction", i.e. empirically drawing the figure on paper and insists on the difference between "constructing in intuition" and "mechanical construction", e.g. on paper. (AA VIII, 191-192, note)

It is not quite clear what Eberhard' point was.[159] He may have thought that Kant's view implies that Appollonius had to demonstrate the construction of the ordinates since conic sections are only possible if the ordinates are possible. If so, then he was of course mistaken since the construction of the cone itself suffices to prove its possibility. However, from Mai-

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157. See Johann August Eberhard, "Ueber die logische Wahrheit oder die transcendentale Gültigkeit der menschlichen Erkenntnis", Philosophisches Magazin, Bd. I (1789), 2. Stück, Halle 1789, pp. 150-174, here: 158-159. Cf. also Eberhard, "Anmerkungen über eine Recension des zqeqten Stück dieses phil. Mag. in der Allg. Litt. Zeit. N. 440. dieses Jahrs", Philosophisches Magazin, Bd. II (1789), 1. Stück, Halle 1789, pp. 29-52, 44-45. See the discussion in Koriako (1999), 253-263.

158. "Über eine Entdeckung..." (1790), AA VIII, 190-192.

159. Koriako ascribes Eberhard an argument we know from Maimon reflections on construction (see above # 3.7), namely that a continuous geometrical object (a segment, a circle, a cone) cannot be strictly constructed. We can construct the *loci geometrici* of these objects, but the continuous spatial objects must be either given or constructed by motion, the latter involving further presuppositions and not accepted in Greek geometry. I find nothing in Eberhard text to support Koriako's interpretation. (Koriako 1999, 253-263)

mon's point of view, Kant does not solve the main difficulty: What is the rule of construction for the cone itself? This question seems to present a major difficulty for Maimon's suggestion to use a conic section to construct a circle. This is so since from a certain point of view this suggestion of an alternative foundation of geometry is a blatant petitio principii and it must either be rejected or else our very notion of "foundations" (and "system") has to be altered.

The petitio principii is obvious: In order to construct the circle and the straight line as conic sections, we need a cone (and a plane). In order to construct a cone, we need a circle and a straight line. One way to construct the cone is to turn a right-angled triangle around its perpendicular side. In fact, this is the *definition* of a cone in *Elements*.[160] Both this and other methods require first a straight line (to construct the triangle) and then motion to construct the basis of the cone (a circle) and its nappes. Again, here too we have to use motion, which is a major characteristic of modern constructions in general and of Kant's in particular. From the point of view of the attempt to construct geometry ab ovo and without motion, this construction by conic sections is a blatant petitio principii, an obvious failure.

However, this is not Maimon's idea of a foundation. It is one of Maimon's principles in all fields of inquiry that neither the most elementary nor the last synthesis can be reached with our finite intellect.

> In our cognition of things we begin hence in the middle and also end in the middle. (Tr, 350)

And specifically with reference to geometrical construction Maimon says:

> The sufficient ground of a thing is the complete concept of its manner of origination [Entstehungsart], which, however, we can only approach,

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160. *Elements* XI, Def. 18. "When a right triangle with one side of those about the right angle remains fixed is carried round and restored again to the same position from which it began to be moved, the figure so comprehended is a cone."

Also Euclid's definition of a sphere involves motion (*Elements* XI, def. 14), namely the roation of a cemicircle around the diameter. However, in Aristotle we find a characterization of the sphere which is analogous to Euclid's definition of the circle and does not involve motion, namely that "its extremity is equally distant from its circle." (Aristotle, De Caelo, II, 14; 297a 24).

Appollonius' definition of the cone involves the rotation of a straight line around the circumference of a given circle.

without ever reaching it, because for the explanation of the mode of origination something that has already arisen [etwas schon Entstandenes] (according to the known axiom that ex nihilo nihil fit) must be presupposed." (GW II, 392, see GW II, 105-106).

We begin with non-elementary objects (here a cone and a plane) given in intuition and reproduce these and other geometrical entities (point, straight line, circle, ellipse, parabola, hyperbola) and gain real insight (of the understanding) into their properties and inter-relations because we produce them as specifically different further determinations of a common general concept (the conic section). The most fundamental objects, however, as well as the farthest consequences that can be drawn from this foundation are so-to-say the vanishing points of geometry, that can be approached ever more but not reached.

We find the same style of inquiry in all areas of Maimon's work and in his philosophy in general. Maimon always proceeds from a given factum and attempts to work his way both "downwards" towards the foundations, and also "upwards" towards further consequences. A major characteristic of his approach is the replacement of elements given in sensuality by determinations of the understanding. Another characteristic feature of Maimon's style is that he warns against leaping from the "middle" to the "vanishing point." Thus he cautions against jumping from ordinary experience to an absolute unity of which all beings would be but finite determinations - this is Spinoza's mistake in philosophy, as it is also the flaw of ancient Jewish monotheism.[161] This of course does not mean that the ideas and the ideals of the understanding and of reason should be given up, but it does mean that they may not be mistaken for given objects of the finite human understanding. Maimon is, therefore, very reserved concerning the philosophical quest for the "highest syntheses" of human thought, as he is also critical of the concentration on the monotheistic "One": finite understanding must put up with its finiteness, begin in the middle and end in the middle and accept the antinomies produced by the duality of the finite and infinite understanding. It is not given to finite man to construct from a point (as Kant attempted), or to reach the highest point of Truth. But at the same time,

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161. On this see my "Salomon Maimon: “Die Philosophical Systems of Theology” "(German), in: Joseph Schwartz & Volkhard Krech (ed), Religious Apologetics - Philosophical Argumentation, Tübingen (Mohr Siebeck) 2004, pp. 87-106

we also cannot give up the notions of these vanishing points. The concept of "Truth" is what makes man an "*animal rationale*," not merely an animal.

7. *Conclusions*

The conclusions from Maimon's thoughts on geometry can be summarized in one sentence, they are, however, of far reaching consequences. In Maimon's view, Kant did not overcome the dilemma presented by Rationalism and Skepticism, by Leibniz and Hume, and it is on principle not possible to resolve it.

Kant suggested that synthetic judgments a priori are possible because we construct concepts in intuition. In geometry, we construct the straight line and the circle and from these all other more complex figures. Kant did not see the problems involved in the construction of the straight line and the circle. In his first introduction (which remained unpublished) to the *Critique of Judgment* (1786), Kant remarked that in elementary geometry we need two "instruments" for the construction of its concepts, namely compass and ruler. However, we do not need "the real instruments". Rather, the instruments signify only the "simplest kinds of representation of the imagination a priori". (AA, XX, 198). Construction itself, the transition from the concept to intuition is not discussed at all, and therefore Kant does not ask what rules replace the instruments of practical construction. He upholds the claim to construct even the straight line and the circle (in pure intuition, not on paper) and takes their construction of to be unproblematic. Maimon showed that this is not the case. We have concepts and we have intuitions, but we cannot bridge the hiatus between them. In the case of the straight line, we do not have a concept at all. In the case of the circle, we have a pure concept of the understanding on the one hand (the Euclidean definition of the circle) and an object in intuition on the other hand; we even have a rule of construction for this object, but this is not implied by the definition . Therefore, we have to add a further step and prove that the construct corresponds to the concept. For this proof we have to introduce further assumptions (invariance of the radius under motion etc.). Kant failed to realize this.

But even if we refuse to allow a construction involving motion into geometry and remain with the Euclidean definition of the circle on the one hand and with a polygon constructed according to this definition and with a circle given in intuition on the other hand, it does not follow that the definition and the polygon have nothing to do with the circle in intuition. After all, we can approximate the circle in construction to an ever greater degree. We may even conclude that concept and object must coincide in infinity, but then the antinomies of infinite arise, as the discussion of the *rota Aristotelis* showed.

Here a typical Maimonean dilemma opens up: We can either uphold the dichotomy between the finite and the infinite - and then we cannot construct continuous objects like the circle; or we may claim that the finite and the infinite converge - and then we cannot uphold the dichotomy between "appearance" and "thing-in-itself" (and must accept the antinomies of the infinite). The example of the circle serves Maimon to argue that the dichotomy of "thing-in-itself" and "appearance" is vacuous:

> "A regular polygon is in relation to the circle (in which it is inscribed, or vice versa) a concept. The circle is in relation to the polygon a thing in itself. Whatever is predicated of the concept of a thing, necessarily applies to the thing itself. But what applies to the thing itself, applies to the concept of the same thing only if and insofar it is identical with it." (GW III, 186)[162]

If we can construct a concept of the understanding in intuition and approximate an object given in intuition until both the constructed and the given coincide, then

> "The things in themselves, and the concept or representation of a thing are objectively one and the same and distinguished only subjectively, i.e. in respect to the completeness of our knowledge." (GW III, 185)

However, this coincidence involves antinomies and, therefore, must not actually occur. The finite intellect cannot overcome the dilemma. Rationalism may be true - and therefore the progress of knowledge would increase the share of the understanding and diminish that of intuition until intuition disappears and the concept of the understanding coincides with the object. This is what the term "infinite understanding" or "God" or "intellectual intuition" stand for. On the other hand, Skepticism may be true - and therefore the progress of knowledge

162. “Ein reguläres Poligon ist in Beziehung auf den Zirkel (in dem oder um den es beschrieben wird) Begriff; der Zirkel hingegen in Beziehung auf das Poligon Ding an sich. Was dem Begriffe eines Dinges zukömmt, kömmt nothwendig dem Dinge selbst zu, was aber dem Dinge selbst zukömmt, kömmt dem Begriffe desselben nur insofern zu, in wiefern er mit ihm identisch ist. “ GW III, 186)

consists in a sedimentation of experience as alleged truths of the understanding and in an extrapolation from our understanding to an alleged infinite understanding which we wish to approach. The concepts of "infinite understanding" or "God" or "intellectual intuition" are then nothing but "the concept of Man [endowed as it were] with infinite perfection" (GM 41), a "regulative concept" which informs us about the conceiving subject, not about its alleged object (GM 51),[163] and therefore does not show that this object exists, but only the path on which we may approach its idea. (GM 53)

Maimon concluded his *Transcendentalphilosophie* with a Talmudic quotation, a simile of this infinite progress of knowledge towards the absolute:

> "Our Talmudists (who certainly expressed at times thoughts worthy of Plato) say: "Scholars (talmidey Chachamim) do not rest, neither in this world nor in the world to come, as it is written: 'They go from strength to strength, every one of them in Zion appeareth before God.'" (Psalms 84, 7(8)).[164]

This conclusion invokes infinite progress in the spirit of Enlightenment. It even promises success. However, the fact that in our cognition of things "we begin ... in the middle and also end in the middle (Tr, 350) has also a much less optimistic connotation: It is an infinite progress towards an end which may not exist at all but be an illusion. It may be that Man is not created in the image of God, but God created in the image of Man. It is possible that the fate of the scholars is the best possible: If they make progress, they have a good reason to hope that human beings indeed belong to the species of *animal rationale* and have preeminence before the beasts. If they ever succeeded in fully achieving their end, they might render all their knowledge analytic, perhaps even prove that it comprises nothing but tautologies, which are

163. "It is hence clear that the aforementioned concept of the infinite is not determining the object (ist nicht ein das Objekt bestimmender Begriff) but determines the conceiver only, i.e. it refers to the conceiver by means of the way it approaches the conceived object (ein regulativer Begriff), namely that it always asks for the cause of the cause already discovered although it cannot ever find all the reasons, i.e. the substance that is an absolute cause, as we explained." (GM 51)

164. GW II, S. 440. Maimon quotes the same phrases in his commentary on Maimonides (GM 40) and added it in the margin to page 10 of his unpublished manuscript Hesheq Shlomo. In the locus quoted above, Maimon significantly mistranslates the term "talmidey Chachamim." Instead of "disciple of the wise" he translates "disciples of wisdom" (Schüler der Weisheit).

not thought at all. Between these frustrating alternatives, the fate of the scholars seems best: their thought is explorative and testifies to their "sparkle of divinity"; it keeps them distant from the end of complete knowledge that at the same time also threatens to collapse all knowledge into a tautology.

The real tragedy consists in this: We will never know which of the alternatives: *animal sensuale sive rationale* is true since the progress of knowledge is infinite. As Maimon showed, every construction and every analysis have to assume something as given and can then proceed a few steps. It is only for a very limited section of the world that we may show that it fits our reason, and it may very well be that the instance that would refute our explanation lurks one step further up or down. We cannot and will never know this.

And yet, the truth about these alternatives entirely changes the nature of our knowledge. Instead of an idea of objective knowledge proper we obtain an idea of an ever more "complete induction" (vollständige Induktion) (Antwort, GW III, 198, 199, 200). Inasmuch as this knowledge is based on intuition, that is not necessarily common to all thinking beings, it is not "objective", but "subjective", not "necessary" but "compulsive". Whereas Kant inferred knowledge's necessity from its universality, Maimon suggests that for all knowledge dependent on intuition (whether empirical or a priori), hence also for geometry, this may be an illusion: what we have here may only be subjective knowledge. This shows most clearly in the specifically geometric "axioms", e.g. that the straight line is shortest between two points or that extended indefinitely parallel lines do not meet. These truths are "imposed" on us (uns aufgedrungen). (GW III, 188)

Because of the compulsion to give our assent to such truths, we ascribe the state of "necessary" knowledge to it, although this is proper to analytic propositions only. Such opaque necessity or complete induction of unfailing universality may reach and be practically "equal" to objective knowledge ("Antwort" GW III, 200). The difference does not (practically!) show, but the implications for the nature of "knowledge" and above all: the consequences concerning the nature of Man are diametrically opposed to each other. There is no doubt that Maimon's sympathies are with the Rationalists but he was honest enough to admit that objectively necessary knowledge may be nothing but an illusion.

Moreover, the dilemma cannot be ignored. Every piece of knowledge involves truth-claims - and these necessarily involve the concept of infinite understanding. The finite and the infinite understanding are always present in human knowledge. Although we cannot prove the truth of either Rational Dogmatism or Empirical Skepticism, we may proceed in the process of acquiring knowledge by introducing the idea of progress:

> "The general antinomy of thought in general evidently involves its resolution: Reason demands that we conceive what is given in an object not as something which by its very nature cannot be changed, but merely as a consequence of the determination (Einschränkung) of our capacity of thinking. Reason thus demands of us progress in infinitum, thus that the [part already] thought augments and the [part] given is reduced to [something] infinitesimally small. The issue is here not how far we can proceed in this, but merely from what perspective we should look at the object in order to correctly conceive it? This (perspective) is nothing else that the idea of a most perfect faculty of thought which we must approach ever more in infinity." (GW III, 193)

This is Maimon the optimist, i.e. the Rationalist. The very same progress distances Maimon from Kant not less than from Empirical Skepticism. If the difference between appearance and thing-in-itself can be rendered infinitesimally small, then also the exclusion of metaphysics from philosophy looses its fundament. Metaphysics is the knowledge of things-in-themselves. If appearance and thing-in-itself are not "perfectly heterogeneous", then metaphysics is the knowledge of the "limits of appearance." Since we cannot know things without knowing their limits (at least not uphold the notion of "truth"), metaphysics is inevitably involved also in finite empirical knowledge (GW III, 200-201)[165]

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165. "Was die letzte Frage anbetrift, nämlich: Wie ist Metaphisik möglich? so muß man erstlich bestimmen, was Metaphisik heist. Ich glaube in der Definizion der Metaphisik mit Herrn Kant übereinstimmen. Nämlich Metaphisik ist die Wissenschaft der Dinge an sich. Ich unterscheide mich von Herrn Kant blos darinn: nach Ihm sind die Dinge an sich die Substrata ihrer Erscheinungen in uns, und mit denselben ganz Heterogen, folglich muß diese Frage unaufgelößt bleiben, indem wir kein Mittel an der Hand haben, die Dinge an sich abstrahirt von unsrer Art von derselben affizirt zu werden, zu erkennen. Nach mir hingegen ist die Erkenntnis der Dinge an sich nichts anders als die vollständige Erkenntnis der Erscheinungen. Die Metaphisik ist also nicht eine Wissenschaft von etwas ausser der Erscheinung, sondern blos von den Gränzen (Ideen) der Erscheinungen selbst, oder von den letzten Gliedern ihrer Reihen. Nun sind zwar diese als Objekte unsrer Erkenntnis unmöglich, sie sind aber mit den Objekten so genau verknüpft, daß

It must be obvious now that there is something amiss in the characterization of Maimon as a Kantian. There is no doubt that Maimon sympathized with Leibniz (and Spinoza) and wished Rational Dogmatism to be true, but as an "Empirical Sketpic" he also maintained that this philosophy may be an illusion and Man not a rational being at all. Maimon did not believe that Kant succeeded in overcoming the alternative between Leibniz and Hume. We have also seen that Maimon successfully interpreted "synthetic judgments a priori" as the "*propria*" of the Aristotelian tradition. In fact, Maimon announced in his last book the "important discovery" that genuine synthetic judgments a priori are convertible without change of quantifier - and thus repeated Ibn Tibbon's criterion of *proprium*, which was known to him.

And yet there is an important difference between Aristotle's and Kant's conceptions. Aristotle does not yet sever in concepts the determinations inferred from those known from experience. "*Idion*" is not introduced as a problem, but as one kind of properties among others. This was different in Kant's time. Kant, too, accepts such concepts as given (the "factum" of pure science and mathematics) However, after Rationalism and Empiricism worked out the strict dichotomy between the understanding and experience, the *proprium* or "synthetic judgment a priori" presented first of all a problem. It seemed that they are impossible: the synthetic cannot be a priori, and the a posteriori cannot be apodictic. The suggestion that synthetic judgments a priori exit, was hence no longer "naive" as in Aristotle, but a very daring philosophical thesis that intended to bridge over the hiatus opened by Leibniz and Hume. Maimon argued that Kant failed. This does not mean that he wished to return to Aristotle. After Leibniz and Hume it was impossible to return to Aristotle, but it was also impossible to accept Kant who did not improve on Aristotle. Maimon seems to believe that Aristotle drew our attention to a special kind of properties and Kant made them the corner stone of his philosophy. Aristotle characterized them, while Kant attempted to account for their possibility pace Leibniz and Hume. Kant failed, and the consequences are that Knowledge in the emphatic sense of the word - synthetic knowledge - became a mystery. Synthetic knowledge seems to exist and be impossible in one. Maimon's example of construction by means of conic sections exemplified such synthetic knowledge a priori. Maimon's Law of Determinability even renders

ohne sie keine vollständige Erkenntnis von den Objekten selbst möglich ist. Wir nähern uns immer zu ihrer Erkenntnis nach dem Grade der Vollständigkeit unsrer Erkenntnis der Erscheinungen." (GW III, 200-201) such knowledge plausible as it spells out the transcendental conditions for synthetic judgments in general. And yet, in the final analysis such knowledge must be impossible. Our present insights suggest that further scrutiny must show that all knowledge is either a priori and analytic or a posteriori and synthetic, but not both. But then "knowledge" is either a collection of tautologies or of mere "habits". We must remain between the horns of these dilemmata.

8. *Appendix: Further Textual Evidence that Maimon Changed the Body of the Transcendentalphilosophie After Receiving Kant's Letter*

My thesis that Maimon edited the *Transcendentalphilosophie* just before its publication is supported by two different kinds of evidence. I argued in this essay that Maimon discovered that and why his proof that the straight line is the shortest between two points was wrong and that he, therefore, changed his views on the nature of geometry and also the text of the *Transcendentalphilosophie*. However, there is also purely historical and textual evidence for this editing, independent of my interpretation of Maimon's views.

My argument was first that Maimon maintains in chapter two that he can prove the postulate and therefore show that it is a priori whereas in the "Kurze Übersicht" he says that it is synthetic a posterioi. These two propositions flatly contradict each other. Maimon's proof in chapter two presupposes that a broken straight line can be replaced for a curve, but his later extensive discussion that a regular polygon is conceptually entirely different from a circle clearly belies this assumption. Hence my suggestion that when realized the latter, he recognized the mistake in the former and changed his mind accordingly. Since it can be shown that Maimon did not yet oppose the definition of a circle to its construction in a paper which appeared in August 1789 (see GW I, 593, 595), and since he answers in the body of *Transcendentalphilosophie* some objections of Kant, it seems safe to argue that Maimon made his discovery and edited the *Transcendentalphilosophie* some time between writing this paper, the reception of Kant's letter (Kant to Herz, May 26, 1789) and the printing of his *Transcendentalphilosophie* which appeared in December 1789.[166]

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166. Samuel Atlas already suggested that Maimon's footnote containing his reservation concerning

Now for the textual evidence: In the body of the text (Tr, 68-70) Maimon answers an objection Kant made against the proof in his letter to Maimon, namely that it proceeds from Wolff's definition of a straight line of which Kant does not approve. Note that neither the objection nor Wolff's definition were mentioned in the *Critique of Pure Reason* or anywhere else in Kant's published work, but only in Kant's letter to Maimon. Thus there can be no doubt that Maimon changed also the body of the text and added footnotes in response to Kant's criticism, as he also edited or added his extensive notes and the long "short summary of the entire work" at the end of the book in which he answered Kant's allegation that he was a "Spinozist."

Moreover, The structure of the printed book does not agree with what Kant says in his letter to Herz. Kant asserts that he read only the first two chapters of the manuscript ("die zwei erste Abschnitte"; ("Abschnitt" is the term appearing in the headline of Maimon's chapters) (AA XI, 49), but he criticizes Maimon's notion of "ideas of the the understanding" (AA XI, 52-53) which Maimon elaborated in what is now chapter three, titled "'Ideas of the Understanding', 'Ideas of Reason' etc."! In the printed version Maimon also answers Kant's criticism in this third chapter. In fact, the chapter is nothing but an answer to Kant's criticism and is very short (GW II, 75-83). It seems plausible that in the sequel of Kant's critique Maimon cut out of chapter two the discussion of ideas of the understanding and pasted it together with his answer to Kant's criticism in a new chapter three.[167] But why? The reason is not difficult to find. Chapter two - by far the longest of the book (Tr 27-74) - contains Maimon's failed proof that the straight line is also the shortest between two points. Immediately after this discussion follows a four pages discussion of Hume, which does not well fit the previous disucssion. It begins with the phrase: "I now come to the question: quid facti?" (Tr, 70)

I suggest that when Maimon recognized that this proof followed from a wrong assump-

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Wolff's definition of the straight line was added in response to Kant's letter to Herz. See *From Critical to Speculative Idealism*. *The Philosophy of Salomon Maimon*, The Hague (Nijhoff) 1964, pp. 138-139, note 21.

167. Achim Engstler, *Untersuchungen zum Idealismus Salomon Maimons*. Stuttgart-Bad Cannstatt: Frommann-Holzboog, 1990, p. 30, note, 12 correctly observes that Maimon's "Kurze Übersicht" and "Anmerkungen und Erläuterungen" at the end of the book show that he edited these after the receipt of Kant's letter. Engstler also remarks there, however, that the ten chapters of the main text remained basically unchanged. This is evidently wrong.

tion, and that he did not succeed in rendering this synthetic judgment a priori analytic, he questioned whether it was a priori at all, hence the quid facti question. Returning to the chapter containing the unfortunate proof, Maimon added his discussion of Hume (pp. 70-75; although he already said on page 57: "To conclude this chapter..."). In fact, chapter two is even much longer than that. If we consider the notes to this chapter (Tr 349-373) the chapter grows from 47 to 71 pages - and, as we know, these notes were added after the receipt of Kant's letter. Finally, in chapter two we find the sole reference to the "Short Overview" at the end of the main text of *Transcendentalphilosophie* (Tr 55), and the relevant portion of this text is another 28 pages long (Tr 168-196). In sum, chapter two is 99 pages long! There is no doubt: This is were the breakthrough took place.

Moreover, in his letter Kant discusses the difference between empirical construction and construction in the imagination (AA XI, 53) - to which Maimon responded in a note to chapter two (Tr 42, note) - as well as whether the construction of a circle requires that all points on the circumference be equidistant from the center (AA XI, 52-53) - to which Maimon answered in the context of the ideas of the understanding in chapter three (Tr, 77-78). I therefore presume that ideas of the understanding and the example of the circle exemplifying this concept were originally introduced in chapter two and then moved to chapter three when Maimon answered Kant's critique. This conjecture is further supported by the fact that the distinction between "formal" and "material" completeness of a concept essential to "ideas of the understanding" is already introduced in chapter two (together with the example of $\sqrt{2}$). However, answering Kant's objections required more space (especially after Maimon also added the discussion of Hume) and Maimon had to move the four pages discussing the ideas of the understanding to a new chapter three, which is therefore little more than eight pages long.

Of course, the question remains why Maimon did not simply delete the failed proof although there can be no doubt at all that he recognized that it was false, not only because he formulated the opposite view in the "Short Overview", but also because he never mentioned the proof again, not even in *Givat Hammore* (which appeared a year later). In this book he maintained that the proposition was a "true belief", not a proven truth.

Maimon's style is in general and in this book in particular anything but orderly. This may be due to his character, to the poor circumstances of his life in this period, or to the incredible wealth of original philosophical ideas in this book which overwhelmed its author. It may, however, be also due to Maimon's cultural heritage. A systematic exposition cannot be found either in the Rabbinic tradition or in the *Guide* of Maimonides. In the introduction to the *Guide*, Maimonides rather discusses possible causes that may be responsible for contradictions in a treatise. The second cause he mentions seems to apply to Maimon's case:

> The author of a particular book has adopted a certain opinion that he later rejects; both his original and his later statements are retained in the book." (*Guide*, Introduction; Pines, 17)

Now, Maimonides takes the examples for contradictions of this kind from the Talmud:

> Contradictions due to the second cause are referred to when they [the sages] say: Rab abandoned this opinion. Raba abandoned that opinion. In such cases an inquiry is made as to which of the two statements is the later one. (*Guide*, Introduction; Pines, 19)

This is what I attempted to do above. However, we should also bear in mind that Maimon was educated in a culture which saw no fault in such contradictions and no reason to eliminate them. There may have even been be a good reason for Maimon to leave the presentation of his former and his later view in the same text: Maimon's early view and his failed proof testified to the sincere attempt to vindicate Rational Dogmatism, such that his later advocacy of Empirical Sketpitcism, (or rather both these views at the same time) won even more credibility. Maimon documented in *Transcendentalphilosophie* the train of thought leading from Rationalism to Empiricism.

Finally, Maimon explicitly argued in *Transcendentalphilosophie* that correctly reasoning from wrong premises was not less true than reasoning from true premisses. In a letter to his friend Lazarus Bendavid he repeated the point and emphasized the worth even of error if it stems from original thought. His example - the squaring of the circle - is exactly the error he later believed to have committed in *Transcendentalphilosophie*:

> Even an error ... at times diminishes only slightly the value of the thought itself, and although a person who believes to have found the

quadrature of the circle, errs in that, I value him much more than the person who merely learned all of mathematics from others without thinking about it for himself even though he did not err.[168]

168. Maimon to Lazarus Bendavid, 7. Februar 1800. In: Jacob Guttmann: Lazarus Bendavid. Seine Stellung zum Judentum und seine literarische Wirksamkeit. In: Monatsschrift für Geschichte und Wissenschaft des Judentums 61 (Neue Folge 25), 1917, S. 207-211. The same praise of "Selbstdenken" can be found also in Mendelssohn: "Ueberhaupt ist es rühmlicher, und der Wahrheit weit erpriesslicher mit Genie von ihr abzuweichen, als dasjenige geistlos zu wiederholen, was andere vor uns schon besser gesagt haben." 56. Literaturbrief. xxx